Quiz 6 Form A
Nov 10, 2017

[10] 1. $\mathcal{L}(u_2(t)(t+5)) = \underline{\quad e^{-2s}(\frac{1}{s^2}+\frac{7}{s}) \quad}$

$$\mathcal{L}(u_2(t)(t+5)) = e^{-2s}\mathcal{L}(t+2+5)) = e^{-2s}\mathcal{L}(t+7(1))) = e^{-2s}(\mathcal{L}(t)+7\mathcal{L}(1))) = e^{-2s}(\tfrac{1}{s^2}+\tfrac{7}{s})$$

[10] 2. Find the eigenvalues and corresponding eigenvectors for $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

An eigenvalue of A is $\underline{\ 0\ }$. The eigenvectors corresponding to this eigenvalue are all nonzero multiples of $\underline{\begin{pmatrix} 2 \\ -1 \end{pmatrix}}$.

An 2nd eigenvalue of A is $\underline{\ 5\ }$. The eigenvectors corresponding to this eigenvalue are all nonzero multiples of $\underline{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}$.

Note it is "obvious" that 0 is an eigenvalue of A since the rows/columns of A are linearly dependent (e.g., row 2 is twice row 1) or equivalently notice $det(A) = 0$.

Since $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, all eigenvectors corresponding to the eigenvalue 0 are all nonzero multiples of $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

To find eigenvalues in general, solve $det(A - rI) = 0$

$$\begin{vmatrix} 1-r & 2 \\ 2 & 4-r \end{vmatrix} = (1-r)(4-r) - 4 = r^2 - 5r + 4 - 4 = r(r-5) = 0$$

Thus the eigenvalues of A are $r = 0,\ 5$

For $r = 5$: $\begin{pmatrix} 1-5 & 2 \\ 2 & 4-5 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}$ and $\begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$