Quiz 5 Form A
Oct 30, 2017

[10] 1. Let y_1 and y_2 be solutions of $ty'' + 4y' + sin(t)y = 0$; $t > 0$. Let $W(t)$ be the Wronskian of $y_1(t)$ and $y_2(t)$. Given that $W(1) = 3$, find $W(t)$.

$W(t) = \underline{\ 3t^{-4}\ }$

$p(t) = \frac{4}{t}$. Thus by Abel's thm $W(t) = ce^{-\int \frac{4dt}{t}} = ce^{-4\int \frac{dt}{t}} = ce^{-4ln|t|} = e^{ln|t|^{-4}} = ct^{-4}$.

Thus $W(t) = ct^{-4}$ for some constant c. Since $W(1) = 3$, $\quad 3 = c(1)^{-4}$. Thus $c = 3$.

Hence $W(t) = 3t^{-4}$

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[10] 2.) Write $y = \sqrt{3}cos(5t) - sin(5t)$ in the form $y = Rcos(\omega t - \delta)$. Determine the period, phase, and amplitude.

y= $\underline{2cos(5t + \frac{\pi}{6})}$

period= $\underline{\ \frac{2\pi}{5}\ }$, phase= $\underline{\ -\frac{\pi}{6}\ }$, and amplitude= $\underline{\ 2\ }$.

amplitude $= R = \sqrt{c_1^2 + c_2^2} = \sqrt{3+1} = \sqrt{4} = 2$

phase $= \delta$ where $tan(\delta) = \frac{c_2}{c_1} = \frac{-1}{\sqrt{3}}$.

Note if you plot $(Rcos(\delta), Rsin(\delta) = (\sqrt{3}, -1)$, you can see the angle the vector $(\sqrt{3}, -1)$ forms with the positive x axis is $-\frac{\pi}{6}$. Thus $\delta = -\frac{\pi}{6}$

Derivation: $Rcos(\omega t - \delta) = Rcos(\delta)R(\omega t) + Rsin(\delta)Rsin(\omega t) = \sqrt{3}cos(5t) - sin(5t)$

Thus $\omega = 5$, period $= \frac{2\pi}{5}$, and $Rcos(\delta) = \sqrt{3}$, $Rsin(\delta) = -1$

Thus $tan(\delta) = \frac{Rsin(\delta)}{Rcos(\delta)} = \frac{-1}{\sqrt{3}}$.

Since $cos(\delta) > 0$ and $sin(\delta) < 0$, δ is in quadrant 4 (i.e, $-\frac{\pi}{2} < \delta < 0$). Thus $\delta = -\frac{\pi}{6}$