Quiz 4 Form B Oct 16, 2017

1. A mass weighing 9 lbs stretches a spring 5 in. The mass is acted on by an external force of $7\sin(3t)$ lbs. The mass is pulled down 2 feet and then set in motion with an upward velocity of 8ft/s. Assume that there is no damping. Note $g = 32ft/s^2$. State the initial value problem that describes the motion of this mass.

IVP:
$$\frac{9}{32}u'' + \frac{108}{5}u = 7sin(3t), \ u(0) = +2, \ u'(0) = -8$$

 $mg = 9.$ Thus $m = \frac{9}{32}.$ $kL = mg.$ Thus $k(5/12) = 9.$ Thus $k = \frac{108}{5}$
 $mu'' + 0u' + ku = 7sin(3t)$

2.) Given that the solution to y'' + y = 0 is $y = c_1 cos(t) + c_2 sin(t)$, what would be a good guess for a non-homogeneous solution to y'' + y = sin(3t)? Note you do not need to solve this differential equation. You also don't need to determine the undetermined coefficients.

Acceptable guess:
$$y = Acos(3t) + Bsin(3t)$$

Best guess: y = Bsin(3t)

Since no y' term, don't need cos term. However, both guesses will give you the correct non-homogeneous solution, so both answers are correct since I didn't ask for the best guess.

3.) Suppose that $y_1(t) = t$ and $y_2(t) = t^2$ are solutions to the differential equation, y'' + p(t)y' + q(t)y = 0. Find the general solution to $y'' + p(t)y' + q(t)y = \frac{1}{t}$

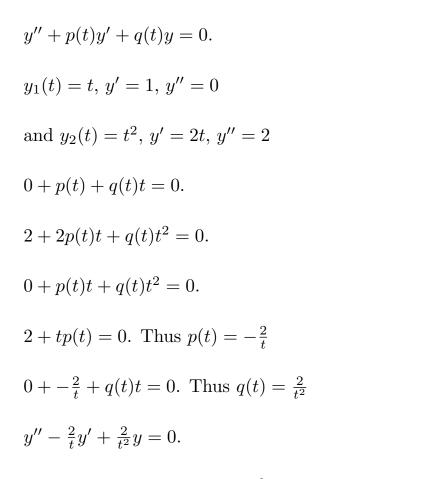
General solution: $y = c_1 t + c_2 t^2 - t \ln |t|$

$$\begin{split} W(t,t^2) &= \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = 2t^2 - t^2 = t^2. \\ \frac{1}{t} \begin{vmatrix} 0 & t^2 \\ 1 & 2t \end{vmatrix} = \begin{vmatrix} 0 & t^2 \\ \frac{1}{t} & 2t \end{vmatrix} = -t \qquad \qquad u_1(t) = \int \frac{g(t)}{a} \frac{W_1}{W} = \int \frac{-t}{t^2} dt = \int \frac{-1}{t} dt = -\ln|t| \\ u_2(t) &= \int \frac{g(t)}{a} \frac{W_2}{W} = \int \frac{1}{t^2} dt = \int t^{-2} dt = -t^{-1} dt = -t^{-1} dt = -t^{-1} dt \end{split}$$

Non-homog: $-tln|t| - t^{-1}t^2 = -tln|t| - t$

General solution: $y = k_1 t + c_2 t^2 - t \ln|t| - t = (k_1 - 1)t + c_2 t^2 - t \ln|t| = c_1 t + c_2 t^2 - t \ln|t|$

FYI:



By Abel's thm, $W(t,t^2)=e^{\int \frac{2}{t}dt}=e^{2ln|t|}=e^{ln|t|^2}=t^2$