Quiz 4 Form A   Oct 16, 2017

1. A mass weighing 5 lbs stretches a spring 8 in. The mass is acted on by an external force of 9sin(2t) lbs. The mass is pulled down 1 foot and then set in motion with an upward velocity of 3ft/s. Assume that there is no damping. Note $g = 32ft/s^2$. State the initial value problem that describes the motion of this mass.

IVP: $\underline{\frac{5}{32}u'' + \frac{15}{2}u = 9sin(2t),\ u(0) = +2,\ u'(0) = -8}$

$mg = 5$. Thus $m = \frac{5}{32}$. $kL = mg$. Thus $k(8/12) = k(2/3) = 5$. Thus $k = \frac{15}{2}$.

$$mu'' + 0u' + ku = 9sin(2t)$$

2.) Given that the solution to $y'' + y = 0$ is $y = c_1cos(t) + c_2sin(t)$, what would be a good guess for a non-homogeneous solution to $y'' + y = cos(2t)$? Note you do not need to solve this differential equation. You also don't need to determine the undetermined coefficients.

Acceptable guess: $\underline{y = Acos(2t) + Bsin(2t)}$

Best guess: $\underline{y = Bcos(2t)}$

Since no y' term, don't need sin term. However, both guesses will give you the correct non-homogeneous solution, so both answers are correct since I didn't ask for the best guess.

3.) Suppose that $y_1(t) = t$ and $y_2(t) = t^2$ are solutions to the differential equation, $y'' + p(t)y' + q(t)y = 0$. Find the general solution to $y'' + p(t)y' + q(t)y = \frac{1}{t}$

General solution: $\underline{y = c_1t + c_2t^2 - tln|t|}$

$$W(t, t^2) = \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = 2t^2 - t^2 = t^2.$$

$$\frac{1}{t}\begin{vmatrix} 0 & t^2 \\ 1 & 2t \end{vmatrix} = \begin{vmatrix} 0 & t^2 \\ \frac{1}{t} & 2t \end{vmatrix} = -t \qquad u_1(t) = \int \frac{g(t)}{a}\frac{W_1}{W} = \int \frac{-t}{t^2}dt = \int \frac{-1}{t}dt = -ln|t|$$

$$\begin{vmatrix} t & 0 \\ 1 & \frac{1}{t} \end{vmatrix} = 1 \qquad u_2(t) = \int \frac{g(t)}{a}\frac{W_2}{W} = \int \frac{1}{t^2}dt = \int t^{-2}dt = -t^{-1}$$

Non-homog: $-tln|t| - t^{-1}t^2 = -tln|t| - t$

General solution: $y = k_1t + c_2t^2 - tln|t| - t = (k_1 - 1)t + c_2t^2 - tln|t| = c_1t + c_2t^2 - tln|t|$

FYI:

$y'' + p(t)y' + q(t)y = 0.$

$y_1(t) = t,\ y' = 1,\ y'' = 0$

and $y_2(t) = t^2,\ y' = 2t,\ y'' = 2$

$0 + p(t) + q(t)t = 0.$

$2 + 2p(t)t + q(t)t^2 = 0.$

$0 + p(t)t + q(t)t^2 = 0.$

$2 + tp(t) = 0.$ Thus $p(t) = -\frac{2}{t}$

$0 + -\frac{2}{t} + q(t)t = 0.$ Thus $q(t) = \frac{2}{t^2}$

$y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 0.$

By Abel's thm, $W(t, t^2) = e^{\int \frac{2}{t}dt} = e^{2ln|t|} = e^{ln|t|^2} = t^2$