Quiz 3 Form B
Sept 25, 2017

1. Sketch the direction field for the autonomous equation $y' = y^2 - 2y - 8$. Find the equilibrium solutions, and classify them as stable or unstable. Sketch the solution with initial value $y(0) = 1$.

[2] Equilibrium solution: _____. Stability of this equilibrium solution _____.

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[2] If $y = \phi(t)$ is the solution to the initial value problem $y' = y^2 - 2y - 8$, $y(0) = 1$, what happens to $y = \phi(t)$ as t goes to $+\infty$?

[4] Sketch of Direction field and solution with initial value $y(0) = 1$:

Choose the best answer for the following problems:

[2] 2.) If $u = y - x$, then

A.) $\frac{dy}{dx} = \frac{du}{dx}$ B.) $\frac{dy}{dx} = \frac{dx}{du}$ C.) $\frac{dy}{dx} = \frac{dx}{dy}$

D.) $\frac{dy}{dx} = \frac{du}{dy}$ E.) $\frac{dy}{dx} = \frac{du}{dx} + 1$ F.) $\frac{dy}{dx} = \frac{du}{dx} + \frac{dx}{du}$

[2] 3.) The integrating factor used to solve the differential equation $\frac{dy}{dx} - \frac{y}{x} = x^2$ is

A.) e^x B.) e^{-x} C.) x D.) $\frac{1}{x}$

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[2] 4.) The solution to the initial value problem $y' = \frac{1-x}{y}$, $y(0) = -\sqrt{3}$ is $y = -\sqrt{-x^2 + 2x + 3}$. State the largest interval on which the solution is defined.

A.) $(-1, \infty)$ B.) $(-1, 3)$ C.) $(3, \infty)$ D.) $(-\infty, 1)$ E.) $(-\infty, 3)$

F.) $[-1, \infty)$ G.) $[-1, 3]$ H.) $[3, \infty)$ I.) $(-\infty, 1]$ J.) $(-\infty, 3]$

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[2] 5.) A tank contains 100 liters of pure water. Saline solution with a variable concentration $c(t) = e^{\frac{-t}{100}}$ grams of salt per liter is pumped into the tank at rate 1 liter per minute. The tank is kept perfectly mixed and also drains at a rate of 1 liter per minute, so the volume stays constant. Write an initial value problem modeling $A(t)$, the amount of grams of salt dissolved in the solution in the tank at time t minutes.

A.) $\frac{dA}{dt} = e^{\frac{-t}{100}} - \frac{Q}{100}, \quad A(0) = 0$ B.) $\frac{dA}{dt} = \frac{Q}{100} - e^{\frac{-t}{100}}, \quad A(0) = 0$

C.) $\frac{dA}{dt} = \frac{Q}{100} + e^{\frac{-t}{100}}, \quad A(0) = 0$ D.) $\frac{dA}{dt} = e^{\frac{-t}{100}} - 100Q, \quad A(0) = 0$

E.) $\frac{dA}{dt} = 100Q - e^{\frac{-t}{100}}, \quad A(0) = 0$ F.) $\frac{dA}{dt} = 100Q + e^{\frac{-t}{100}}, \quad A(0) = 0$

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[2] 6.) Note this is a 3.1 problem. The general solution to $y'' - 2y' - 8y = 0$ is

A.) $y = c_1e^{2t} + c_2e^{4t}$ B.) $y = c_1e^{2t} + c_2e^{-4t}$ C.) $y = c_1e^{-2t} + c_2e^{4t}$

D.) $y = c_1e^{-2t} + c_2e^{-4t}$ E.) $y = c_1e^{2t} + c_2te^{2t}$ F.) $y = c_1cos(2t) + c_2sin(2t)$