$y \xrightarrow{\mathcal{L}} \mathcal{L}(y) = F(s)$

$y = f(t) \xrightarrow{\mathcal{L}} \mathcal{L}(f(t)) = F(s)$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 31
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 31
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 7
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a \rvert$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a \rvert$	Sec. 6.1; Prob. 7
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 25
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2; Cor. 6.2.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 29

$y = f(t)$

$y(0) = f(0)$

$y^{(n)} \xrightarrow{\mathcal{L}} s^n \mathcal{L}(y) - s^{n-1}y(0) - s^{n-2}y'(0) \cdots - s y^{(n-2)}(0) - y^{(n-1)}(0)$

Def $$\boxed{\mathcal{L}(f'(t)) = \int_0^\infty e^{-st} f'(t)dt = \lim_{A\to\infty} \int_0^A e^{st} f'(t)dt}$$

Integration by parts: Let $u = e^{-st}$ $\quad dv = f'(t)dt$

Then $du = -se^{st}dt$ $\quad v = f(t)$

Goal: to get rid of derivative

$$\int_0^A e^{-st} f'(t)dt = e^{-st}f(t)|_0^A + \int_0^A [+se^{-st}f(t)]dt$$

$$= e^{-sA}f(A) - f(0) + s\int_0^A e^{-st}f(t)dt$$

s constant wrt dt

Thus $\mathcal{L}(f'(t)) = \lim_{A\to\infty} \int_0^A e^{st} f'(t)dt$

So can pull it out of integral

$$= \lim_{A\to\infty} \left(e^{-sA}f(A) - f(0) + s\int_0^A e^{-st}f(t)dt\right)$$

under appropriate hypotheses

$$= 0 - f(0) + s\int_0^\infty e^{-st}f(t)dt$$

$$= 0 - f(0) + s\mathcal{L}(f(t))$$

$$\boxed{\mathcal{L}(f'(t)) = s\mathcal{L}(f(t)) - f(0)}$$

Thus $\mathcal{L}(f'(t)) = s\mathcal{L}(f(t)) - f(0)$

and $\mathcal{L}(f''(t)) = s\mathcal{L}(f'(t)) - f'(0) = s[s\mathcal{L}(f(t)) - f(0)] - f'(0)$

$$= s^2\mathcal{L}(f(t)) - sf(0) - f'(0)$$

and $\mathcal{L}(f'''(t)) = s\mathcal{L}(f''(t)) - f''(0)$

$$= s[s^2\mathcal{L}(f(t)) - sf(0) - f'(0)] - f''(0)$$

$$= s^3\mathcal{L}(f(t)) - s^2f(0) - sf'(0) - f''(0)$$

etc. And thus

$$\mathcal{L}(f^{(n)}(t)) = s^n\mathcal{L}(f(t)) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

Formula 18