Solving polynomial equations:

$$\text{Example: } r^3 + r^2 + 3r + 10 = 0$$

Plug in $r = \pm 1, \pm 2, \pm 5, \pm 10$ to see if any of these are solns:

$$(\pm 1)^3 + (\pm 1)^2 + 3(\pm 1) + 10 \neq 0$$

$$(\pm 2)^3 + (\pm 2)^2 + 3(\pm 2) + 10 \; ? =? 0$$

$$(-2)^3 + (-2)^2 + 3(-2) + 10 = -8 + 4 - 6 + 10 = 0$$

Thus $(r - (-2))$ is a factor of $r^3 + r^2 + 3r + 10$

Hence $r^3 + r^2 + 3r + 10 = (r+2)(r^2 + \underline{\;-1\;}\, r + 5)$ $= r^3 + r^2 + 3r + 10$ ✓

Check your factoring!!!

To find the coefficient of r in the above, you can do so by (1) long division, (2) inspection, (3) using variable x

$$r^3 + r^2 + 3r + 10 = (r+2)(r^2 + \underline{\;x\;}\, r + 5)$$

$$(r+2)(r^2 + \underline{\;x\;}\, r + 5) = r^3 + (2+x)r^2 + (2x+5)r + 10$$

$$r^3 \quad + \quad r^2 \quad + \quad 3r \quad + \quad 10$$

Thus $2 + x = 1$ and hence $x = -1$

Check: $2x + 5 = 2(-1) + 5 = 3$

Hence $r^3 + r^2 + 3r + 10 = (r+2)(r^2 - r + 5) = 0$

Thus $r = -2, \frac{1 \pm \sqrt{1-20}}{2}$. Thus $r = -2, \frac{1 \pm i\sqrt{19}}{2}$.

$r^n + b = 0$

In special cases, you can use the unit circle.

Ex: $r^4 + 1 = 0$ implies $\qquad e^{i\theta} = \cos\theta + i\sin\theta$

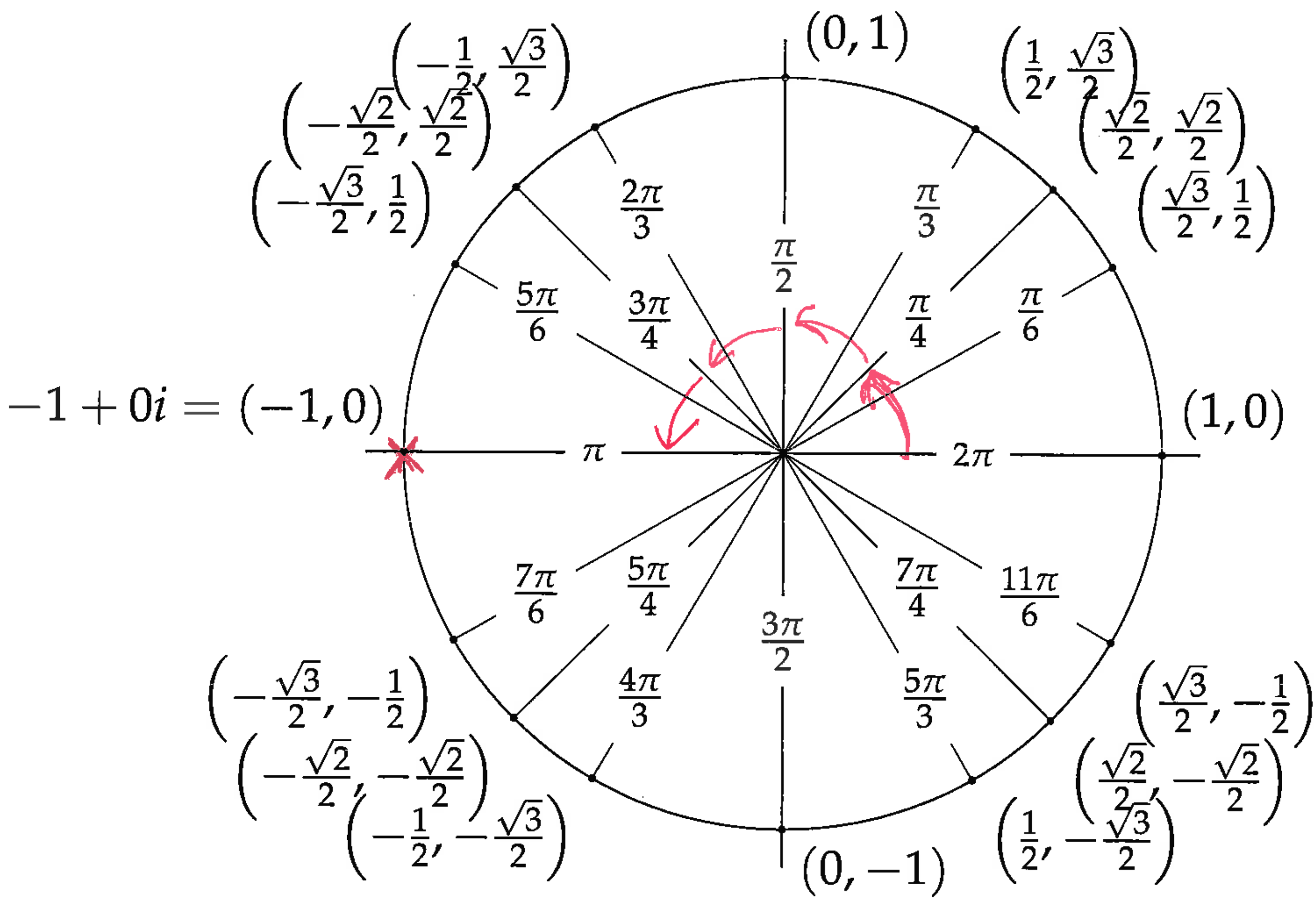
$$r = (-1)^{\frac{1}{4}} = (-1+0i)^{\frac{1}{4}} = (e^{i\pi})^{\frac{1}{4}} = (e^{i(\pi+2\pi k)})^{\frac{1}{4}}$$

$$k=0: \qquad e^{\frac{i\pi}{4}} = cos(\tfrac{\pi}{4}) + isin(\tfrac{\pi}{4}) = \tfrac{\sqrt{2}}{2} + i\tfrac{\sqrt{2}}{2}$$

$$k=1: \qquad e^{\frac{3i\pi}{4}} = cos(\tfrac{3\pi}{4}) + isin(\tfrac{3\pi}{4}) = -\tfrac{\sqrt{2}}{2} + i\tfrac{\sqrt{2}}{2}$$

$$k=2: \qquad e^{\frac{5i\pi}{4}} = cos(\tfrac{5\pi}{4}) + isin(\tfrac{5\pi}{4}) = -\tfrac{\sqrt{2}}{2} - i\tfrac{\sqrt{2}}{2}$$

$$k=3: \qquad e^{\frac{7i\pi}{4}} = cos(\tfrac{7\pi}{4}) + isin(\tfrac{7\pi}{4}) = \tfrac{\sqrt{2}}{2} - i\tfrac{\sqrt{2}}{2}$$



4.2 / 4.3

Solve $y^{(6)} - 2y''' + y = t$

Step 1: Solve homog

Guess $y = e^{rt}$

$$r^6 - 2r^3 + 1 = 0$$

$$(r^3)^2 - 2r^3 + 1 = 0$$

$$\boxtimes^2 - 2\boxtimes + 1 = 0$$

$$(\boxtimes - 1)^2 = 0$$

$$(r^3 - 1)^2 = 0$$

$$r^3 = 1$$

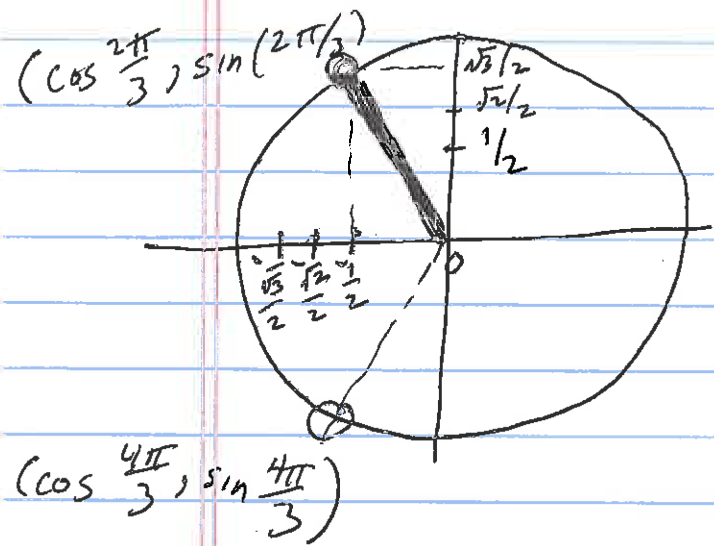
$$r = (1)^{1/3} = (e^{i0})^{1/3} = \left[e^{i(0+2\pi k)}\right]^{\frac{1}{3}}$$

want complex roots

$k=0: \; e^{i0/3} = e^{0} = 1$

$k=1: \; e^{(0+2\pi i/3)} = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$

$= -\frac{1}{2} + \frac{i\sqrt{3}}{2}$



$k=2: \; e^{i(0+4\pi)/3} = e^{4\pi i/3} = \cos\left(\frac{4\pi}{3}\right) + i\sin\frac{4\pi}{3}$

$= -\frac{1}{2} - \frac{i\sqrt{3}}{2}$

$\Rightarrow r = 1, \; -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

note: complex sol'ns always come in complex conjugate pairs

These are repeated roots
They all occur twice

General soln $y = c_1 e^{t} + c_2 e^{-\frac{t}{3}}\cos\left(\frac{\sqrt{3}}{2}t\right) + c_3 e^{-t/3}\sin\left(\frac{\sqrt{3}}{2}t\right)$

$+ c_4 t e^{t} + c_5 t e^{-t/3}\cos\left(\frac{\sqrt{3}}{2}t\right) + c_3 t e^{-t/3}\sin\left(\frac{\sqrt{3}}{2}t\right)$

Step 2: Find one nonhomog soln

$$y^{(6)} - 2y''' + y = t$$

Guess $y = At + B$

$\Rightarrow y' = A, \quad y'' = 0 = y''' = y^{iv} = y^{v} = y^{vi}$

Plug in: $0 - 2(0) + At + B = t$

$\Rightarrow A = 1, \; B = 0$

$\Rightarrow$ General nonhomog soln:

$$\boxed{y = c_1 e^{t} + c_2 e^{-t/2} \cos\left(\tfrac{\sqrt{3}}{2} t\right) + c_3 e^{-t/2} \sin\left(\tfrac{\sqrt{3}}{2} t\right) + t\left[c_4 e^{t} + e^{-t/2}\left[c_5 \cos\left(\tfrac{\sqrt{3}}{2} t\right) + c_6 \sin\left(\tfrac{\sqrt{3}}{2} t\right)\right]\right] + t}$$