homog

Thm: Suppose $c_1\phi_1(t) + c_2\phi_2(t)$ is a general solution to

$$ay'' + by' + cy = 0,$$

If ψ is a solution to

nonhom

$$ay'' + by' + cy = g(t) \;[*],$$

Then $\boxed{\psi + c_1\phi_1(t) + c_2\phi_2(t)}$ is also a solution to [*].

nonh homog

Moreover if γ is also a solution to [*], then there exist constants c_1, c_2 such that

$$\boxed{\gamma = \psi + c_1\phi_1(t) + c_2\phi_2(t)}$$

nonhom homog

Or in other words, $\psi + c_1\phi_1(t) + c_2\phi_2(t)$ is a general solution to [*].

Proof:

Define $L(f) = af'' + bf' + cf$.

Recall L is a linear function.

Let $h = c_1\phi_1(t) + c_2\phi_2(t)$. Since h is a solution to the differential equation, $ay'' + by' + cy = 0$,

$ah'' + bh' + ch = 0$

Since ψ is a solution to $ay'' + by' + cy = g(t)$,

$a\psi'' + b\psi' + c\psi = g$

We will now show that $\psi + \overbrace{c_1\phi_1(t) + c_2\phi_2(t)}^{h} = \psi + h$ is also a solution to [*].

$a(\psi+h)'' + b(\psi+h)' + c(\psi+h)$

$(a\psi'' + b\psi' + c\psi) + (ah'' + bh' + ch)$

$= g + 0 = g$

Since γ a solution to $ay'' + by' + cy = g(t)$,

We will first show that $\gamma - \psi$ is a solution to the differential equation $ay'' + by' + cy = 0$.

Since $\gamma - \psi$ is a solution to $ay'' + by' + cy = 0$ and

$c_1\phi_1(t) + c_2\phi_2(t)$ is a general solution to

$$ay'' + by' + cy = 0,$$

there exist constants c_1, c_2 such that

$$\gamma - \psi = \underline{\hspace{8cm}}$$

Thus $\gamma = \psi + c_1\phi_1(t) + c_2\phi_2(t)$.

11.) $y'' - 4y' - 5y = 4sin(3t) + 5cos(3t)$

12.) $y'' - 4y' - 5y = 4e^{-t}$

To solve $ay'' + by' + cy = g_1(t) + g_2(t) + ...g_n(t)$ [**]

Step 1.) Find the general solution to $ay'' + by' + cy = 0$:

solve homog $\quad c_1\phi_1 + c_2\phi_2$

2.) For each g_i, find a solution to $ay'' + by' + cy = g_i$:

$$\psi_i$$

This includes plugging guessed solution ψ_i into $ay'' + by' + cy = g_i$.

The general solution to [**] is

$$\underbrace{c_1\phi_1 + c_2\phi_2}_{0} + \psi_1 + \psi_2 + ...\psi_n$$

$0 + g_1 + g_2 + g_2$

3.) If initial value problem:

Once general solution is known, can solve initial value problem (i.e., use initial conditions to find c_1, c_2).

can break into simpler problems