2.) Circle the differential equation whose direction field is given below:

A) $y' = t^2$ **B)** $y' = \frac{1}{2}$

C) $y' = 1$ **D)** $y' = -1$

E) $y' = y + 1$ **F)** $y' = y - 2$

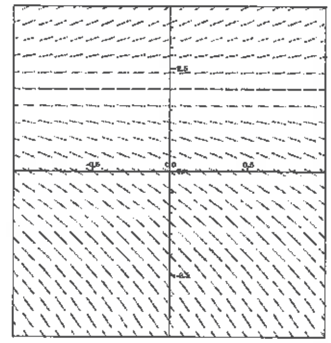
G) $y' = (y+1)(y-2)$ H) $\cancel{y' = (y+1)^2(y-2)^2}$

I) $\cancel{y' = (y+1)(y-2)^2}$ **J)** $y' = (y+1)^2(y-2)$

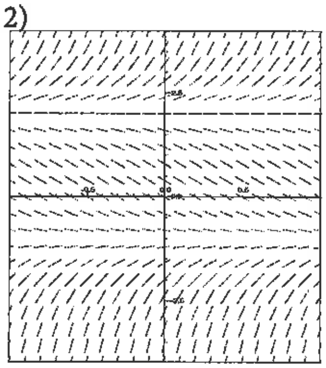
K) $y = -\frac{t}{y}$

J

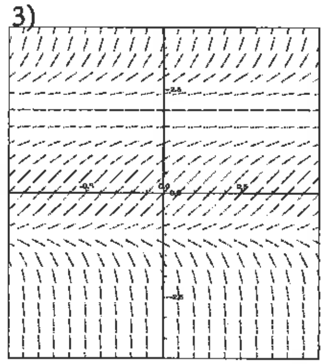
1)



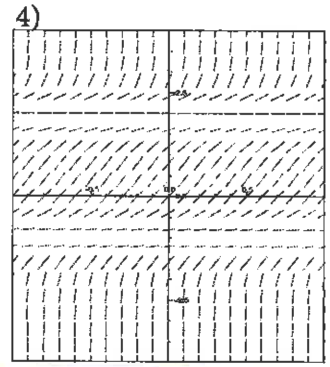
2)



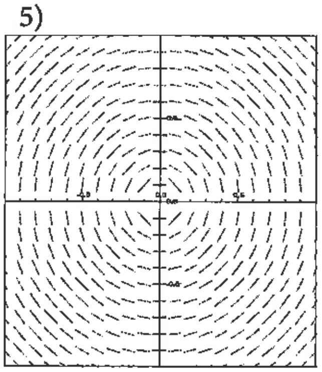
3)



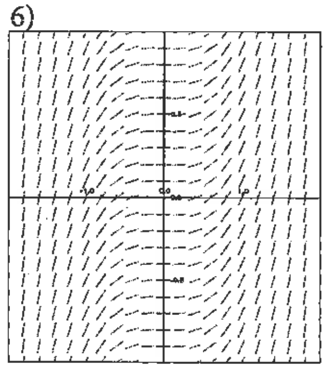
4)



5)

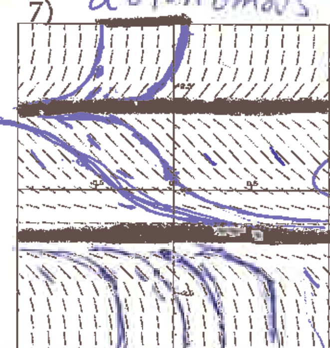


6)



autonomous $y' = f(y)$

7)



2

-1

$+$

$-$

$-$

today's quiz

Equil soln

$y = 2$ unstable

$y = -1$ semi stabl

Proof outline of thm 2.4.2: existance & uniqueness

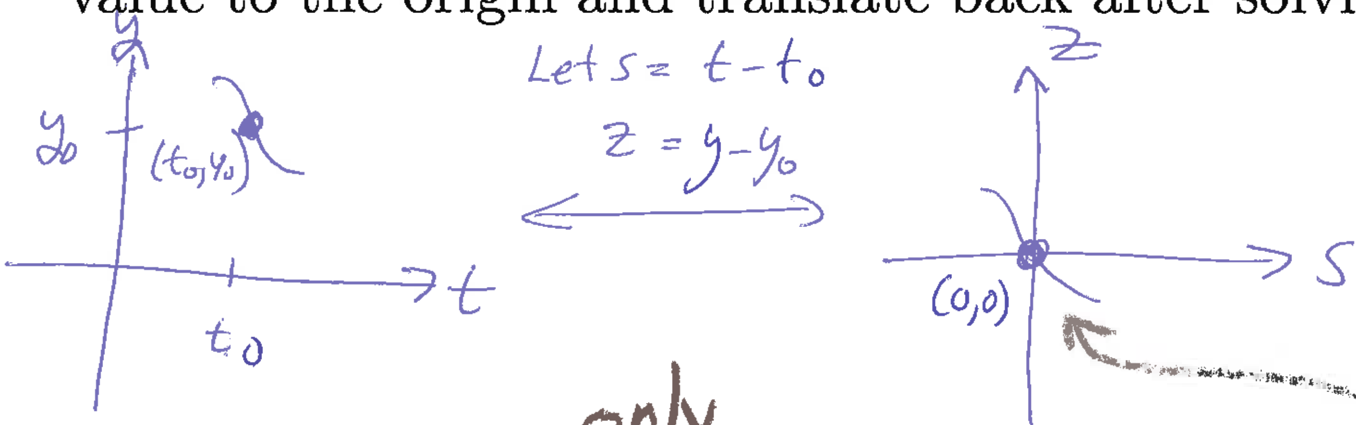
2.8: Approximating solution using

Method of Successive Approximation

(also called Picard's iteration method).

IVP: $y' = f(t, y)$, $y(t_0) = y_0$.

Note: Can always translate IVP to move initial value to the origin and translate back after solving:



Hence for simplicity only **in section 2.8**, we will assume initial value is at the origin: $y' = f(t, y)$, $y(0) = 0$.

Thm 2.4.2: Suppose the functions $z = f(t, y)$ and $z = \frac{\partial f}{\partial y}(t, y)$ are continuous on $(a, b) \times (c, d)$ and the point $(t_0, y_0) \in (a, b) \times (c, d)$,

then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$y' = f(t, y), \quad y(t_0) = y_0.$$

Thm 2.8.1 is translated to origin version of Thm 2.4.2:

Thm 2.8.1: Suppose the functions

$z = f(t, y)$ and $z = \frac{\partial f}{\partial y}(t, y)$ are continuous for all t in $(-a, a) \times (-c, c)$,
then there exists an interval $(-h, h) \subset (-a, a)$ such that there exists a unique function $y = \phi(t)$ defined on $(-h, h)$ that satisfies the following initial value problem:

$$y' = f(t, y), \quad y(0) = 0.$$

Proof outline (note this is a constructive proof and thus the proof is very useful).

Given: $y' = f(t, y)$, $y(0) = 0$ Eqn (*)
f, $\partial f / \partial y$ continuous $\forall (t, y) \in (-a, a) \times (-b, b)$.

Then $y = \phi(t)$ is a solution to (*) iff

$\phi'(t) = f(t, \phi(t)), \quad \phi(0) = 0$ iff

$\int_0^t \phi'(s)ds = \int_0^t f(s, \phi(s))ds, \quad \phi(0) = 0$ iff

$\phi(t) = \phi(t) - \phi(0) = \int_0^t f(s, \phi(s))ds$

Thus $y = \phi(t)$ is a solution to (*)

iff $\phi(t) = \int_0^t f(s, \phi(s))ds$

FYI (ie not on quizzes/exam):

Defn: $\sum_{k=0}^{\infty} a_k x^k = \lim_{n\to\infty} \sum_{k=0}^{n} a_k x^k$

$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots.$$

Taylor's Theorem: If f is analytic at 0, then for small x (i.e., x near 0),

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$$= f(0) + f'(0)x + \tfrac{f''(0)}{2} x^2 + \tfrac{f'''(0)}{6} x^3 + \ldots$$

Example:

$e^t = \sum_{k=0}^{\infty} \frac{t^k}{k!}$ and thus $e^{bt} = \sum_{k=0}^{\infty} \frac{b^k t^k}{k!}$ for t near 0.

$$\phi_n(t) = \sum_{k=2}^{n} \frac{2^{k-2}}{k!} t^k$$

Thus $\phi(t) = \lim_{n\to\infty} \phi_n(t) = \sum_{k=2}^{\infty} \frac{2^{k-2}}{k!} t^k = \frac{1}{4} \sum_{k=2}^{\infty} \frac{2^k}{k!} t^k$

$$= \frac{1}{4}(\qquad - \qquad - \qquad)$$