FYI

4.) Circle the general solution to the differential equation whose direction field is given below:

~~A) $y = t + C$~~

~~B) $y = t^2 + C$~~

~~C) $y = e^t + C$~~

D) $y = Ce^t + t + 1$

E) $y = Ce^t$

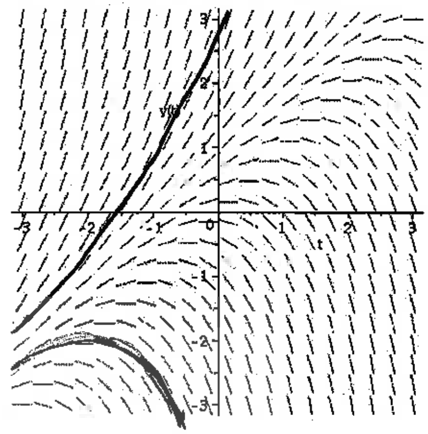
~~F) $y = e^t + t + C$~~

~~G) $y = ln(t) + C$~~

~~H) $y = C$~~

~~I) $y = sin(t) + C$~~

J) ~~$y = cos(t) + C$~~



5.) Which of the following could be the general solution to the differential equation whose direction field is given below:

A) $y = t + C$

B) $y = t^2 + C$

C) $y = e^t + C$

D) $y = \frac{(t-1)^3}{3} + C$

E) $y = Ce^t$

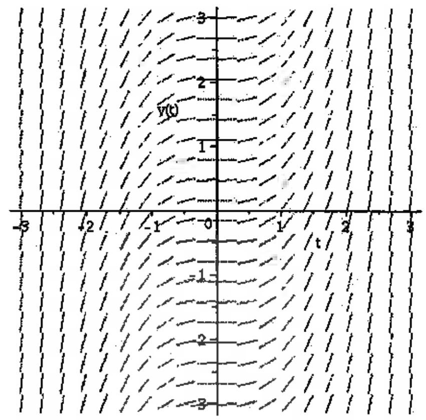
F) $y = \frac{t^3}{3} + C$

G) $y = ln(t) + C$

H) $y = C$

I) $y = \frac{Ct^3}{3}$

J) $y = \frac{C(t-1)^3}{3}$



6.) Circle the differential equation whose direction field is given below:

~~A) $y' = t^2$~~

B) $y' = y + 3$

~~C) $y' = e^t$~~

D) $y' = t + 1$

~~E) $y' = t - y$~~

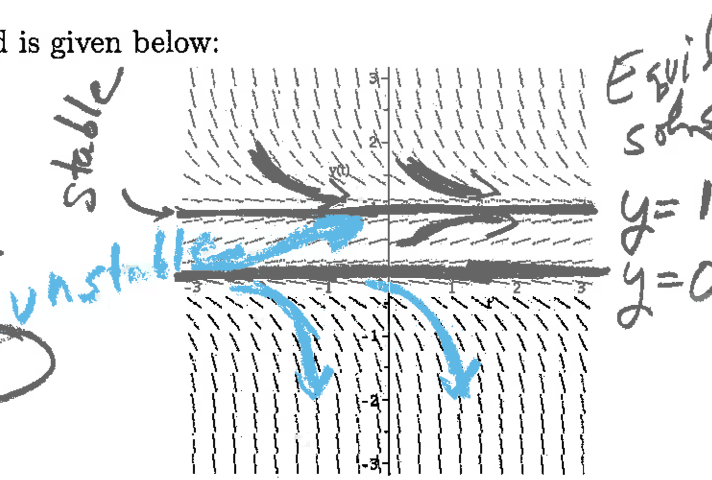
~~F) $y' = y - t$~~

G) $y' = (1 + y)(1 - y)$

H) $y' = y(1 + y)$

~~I) $y' = t(1 - t)$~~

J) $y' = y(1 - y)$



2.5 Autonomous

$y' = f(y)$

slope y' only depends on y

Section 2.5 Autonomous equations: $y' = f(y)$

If given either differential equation $y' = f(y)$

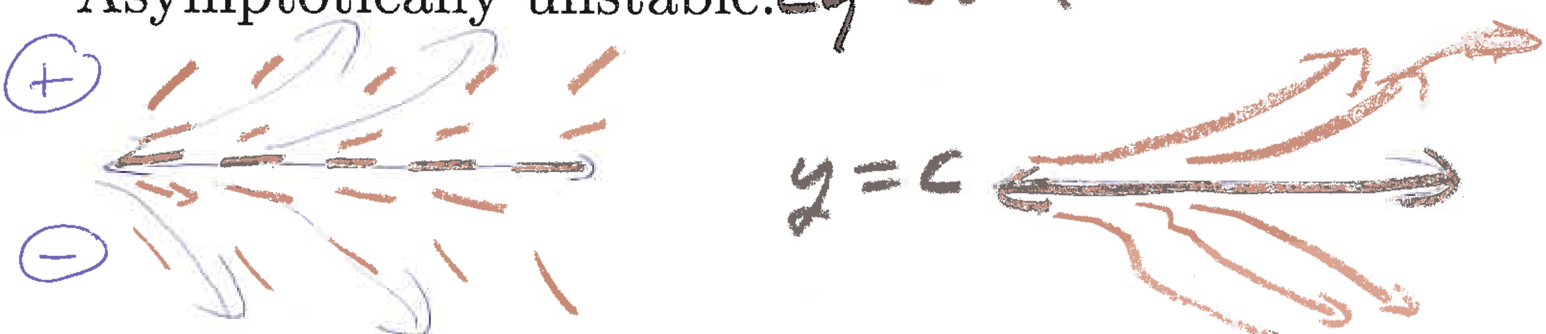
$y = c \iff y' = 0 \iff f(y) = 0$ OR direction field:

Find equilibrium solutions and determine if stable, unstable, semi-stable.

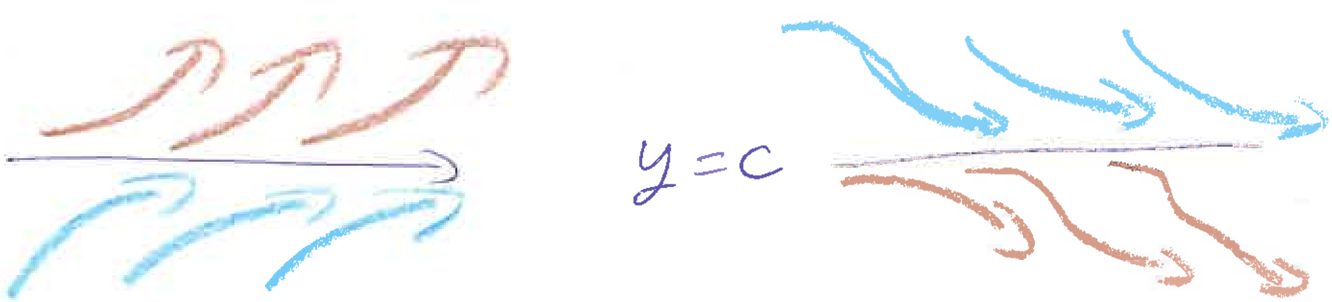
Understand what the above means.

Asymptotically stable: eq soln

Asymptotically unstable: eq soln



Asymptotically semi-stable: eq soln



Section 2.5: Autonomous equations: $y' = f(y)$

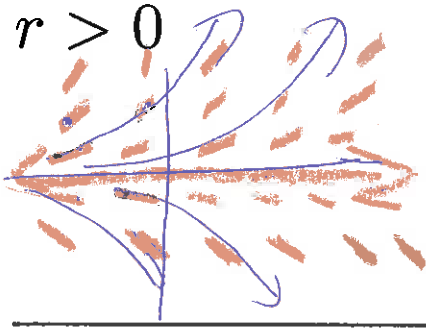
Example: Exponential Growth/Decay

Example: population growth/radioactive decay

$$y' = ry,\ y(0) = y_0 \text{ implies } y = y_0 e^{rt}$$

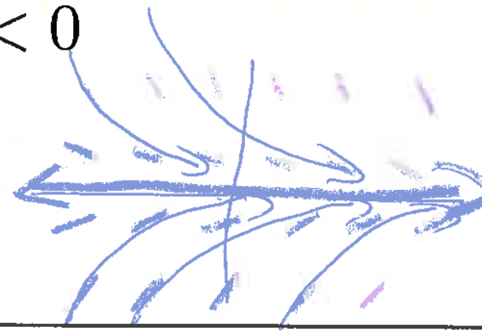
$y = 0$ unstable eqlb soln

$r > 0$



$y = 0$ STABLE eqlb soln

$r < 0$



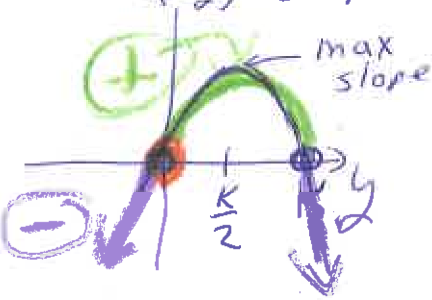
Example: Logistic growth: $y' = h(y)y$

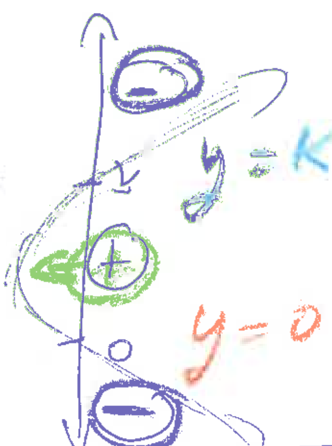
Example: $y' = r(1 - \frac{y}{K})y$

$f(y) = r(1 - \frac{y}{K})y$

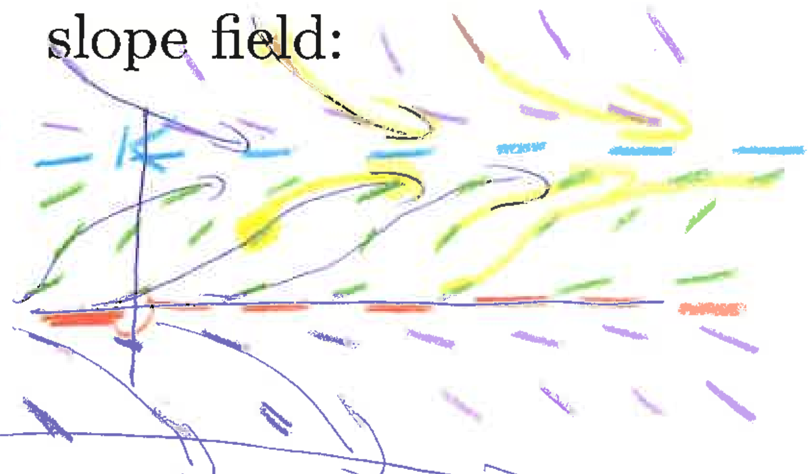
y vs $f(y)$

$f(y) = $ slope





slope field:



Equilibrium solutions: $y = 0$ unstable & $y = K$ stable

As $t \to \infty$, if $y_0 > 0$, $y \to K$

$y(t_0) = y_0$

Solution: $y = \frac{y_0 K}{y_0 + (K - y_0)e^{-rt}}$

Special cases: 2.4 uniqueness existence

Suppose f is cont. on (a, b) and the point $t_0 \in (a, b)$,
Solve IVP: $\frac{dy}{dt} = f(t)$, $y(t_0) = y_0$

$$dy = f(t)dt$$

$$\int dy = \int f(t)dt$$

$y = F(t) + C$ where F is any anti-derivative of F.

Initial Value Problem (IVP): $y(t_0) = y_0$

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$

Hence unique solution (if domain connected) to IVP:

$$y = F(t) + y_0 - F(t_0)$$

First order linear differential equation:

Thm 2.4.1: If p and g are continuous on (a, b) and the point $t_0 \in (a, b)$, then there exists a unique function $y = \phi(t)$ defined on (a, b) that satisfies the following initial value problem:

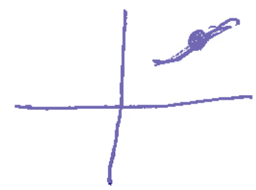
$$y' + p(t)y = g(t), \quad y(t_0) = y_0.$$

More general case (but still need hypothesis)

Thm 2.4.2: Suppose the functions
$\boxed{z = f(t,y)}$ and $\boxed{z = \frac{\partial f}{\partial y}(t,y)}$ are continuous on $(a,b) \times (c,d)$ and the point $(t_0, y_0) \in (a,b) \times (c,d)$,

then there exists an interval $(t_0 - h, t_0 + h) \subset (a,b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$\boxed{y' = f(t,y),} \quad y(t_0) = y_0.$$



Section 2.4 example: $\frac{dy}{dt} = \frac{1}{(1-t)(2-y)}$ ← not linear so can only use Thm 2.4.2 to see if you know anything about existence & uniqueness

$F(y,t) = \frac{1}{(1-t)(2-y)}$ is continuous for all $t \neq 1$, $y \neq 2$

$$\frac{\partial F}{\partial y} = \frac{\partial\left(\frac{1}{(1-t)(2-y)}\right)}{\partial y} = \frac{1}{(1-t)}\frac{\partial(2-y)^{-1}}{\partial y} = \frac{1}{(1-t)(2-y)^2}$$

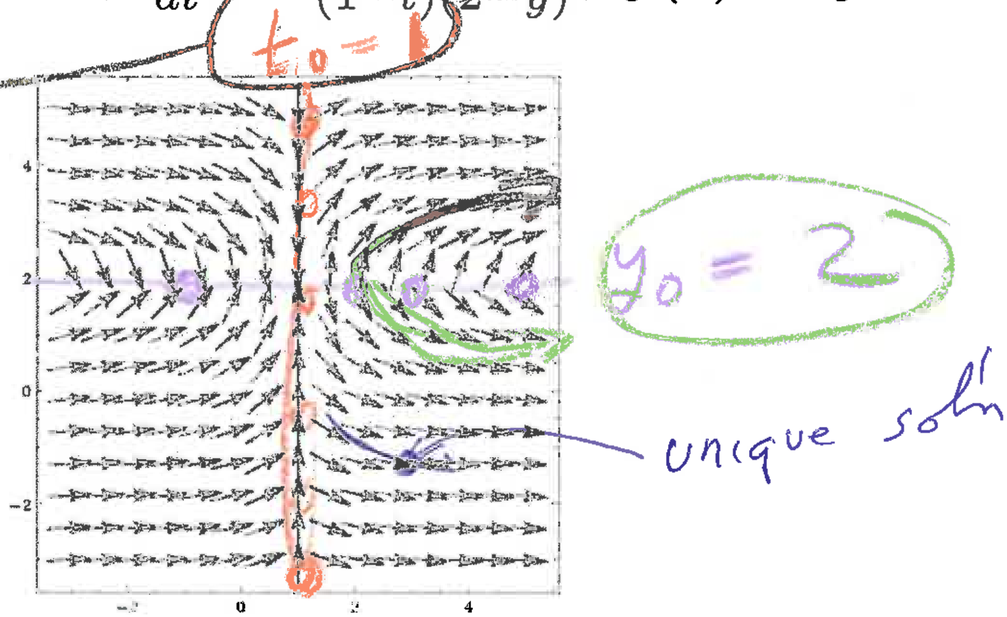
$\frac{\partial F}{\partial y}$ is continuous for all $t \neq 1$, $y \neq 2$

Thus the IVP $\frac{dy}{dt} = \frac{1}{(1-t)(2-y)}$, $y(t_0) = y_0$ has a unique solution if $t_0 \neq 1$, $y_0 \neq 2$.

Don't know about initial values $(1, y_0)$
$(t_0, 2)$

Note that if $y_0 = 2$, $\frac{dy}{dt} = \frac{1}{(1-t)(2-y)}$, $y(t_0) = 2$ has two solutions if $t_0 \neq 1$

Note that if $t_0 = 1$, $\frac{dy}{dt} = \frac{1}{(1-t)(2-y)}$, $y(1) = y_0$ has no solutions.



$$(1, 1/((1-t)(2-y)))/sqrt(1 + 1/((1-t)(2-y))^2)$$

Solve via separation of variables:

$\int (2-y)dy = \int \frac{dt}{1-t}$

$2y - \frac{y^2}{2} = -ln|1-t| + C$

$y^2 - 4y - 2ln|1-t| + C = 0$

$y = \frac{4 \pm \sqrt{16 + 4(2ln|1-t| + C)}}{2} = 2 \pm \sqrt{4 + 2ln|1-t| + C}$

$$y = 2 \pm \sqrt{2ln|1-t| + C}$$