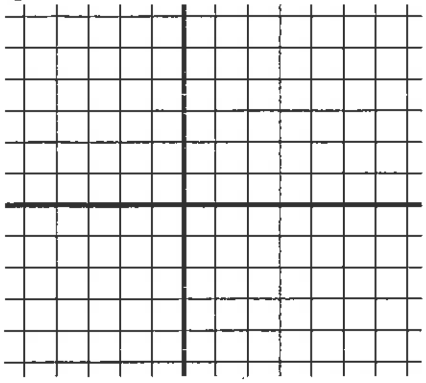
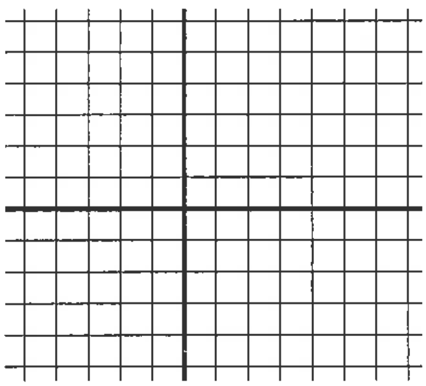
8.1 supplemental HW

1.) For each of the following differential equations (i) draw its direction field; (ii) sketch the solution of the direction field that passes through the point (-2, 1); (iii) state the general solution to the differential equation.

a.) $y' = 0$



b.) $y' = -1$



2.) Circle a solution to the differential equation whose direction field is given below:

FYI

A) $y = t^2$ | B) $y = \frac{1}{2}t + 1$

C) $y = e^t$ | D) $y = t + 1$

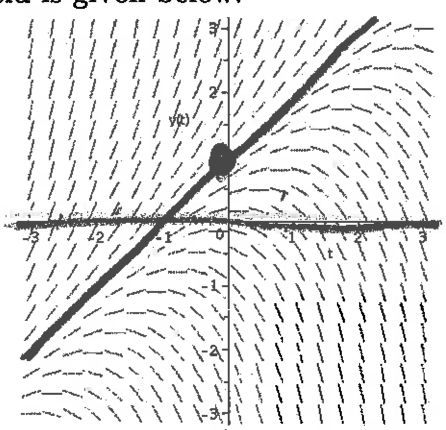
E) $y = -2e^t$ | F) $y = 2t + 1$

G) $y = ln(t)$ | H) $\cancel{y = 0}$

I) $y = sin(t)$ | J) $\cancel{y = cos(t)}$

IVP

IVP Soln



3.) Circle the differential equation whose direction field is given below:

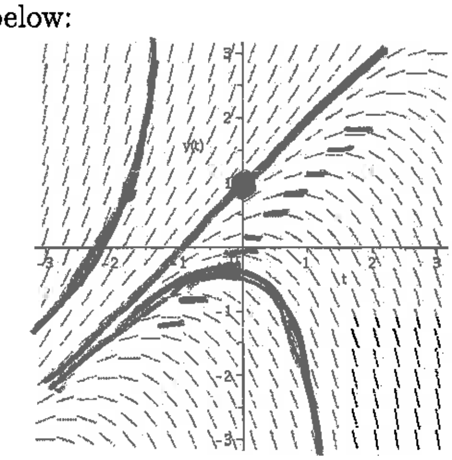
A) $y' = t^2$ | B) $y' = \frac{1}{2}t + 1$

C) $y' = e^t$ | D) $y' = t + 1$

E) $y' = -2e^t$ | F) $y' = y - t$

G) $y' = ln(t)$ | H) $y' = 0$

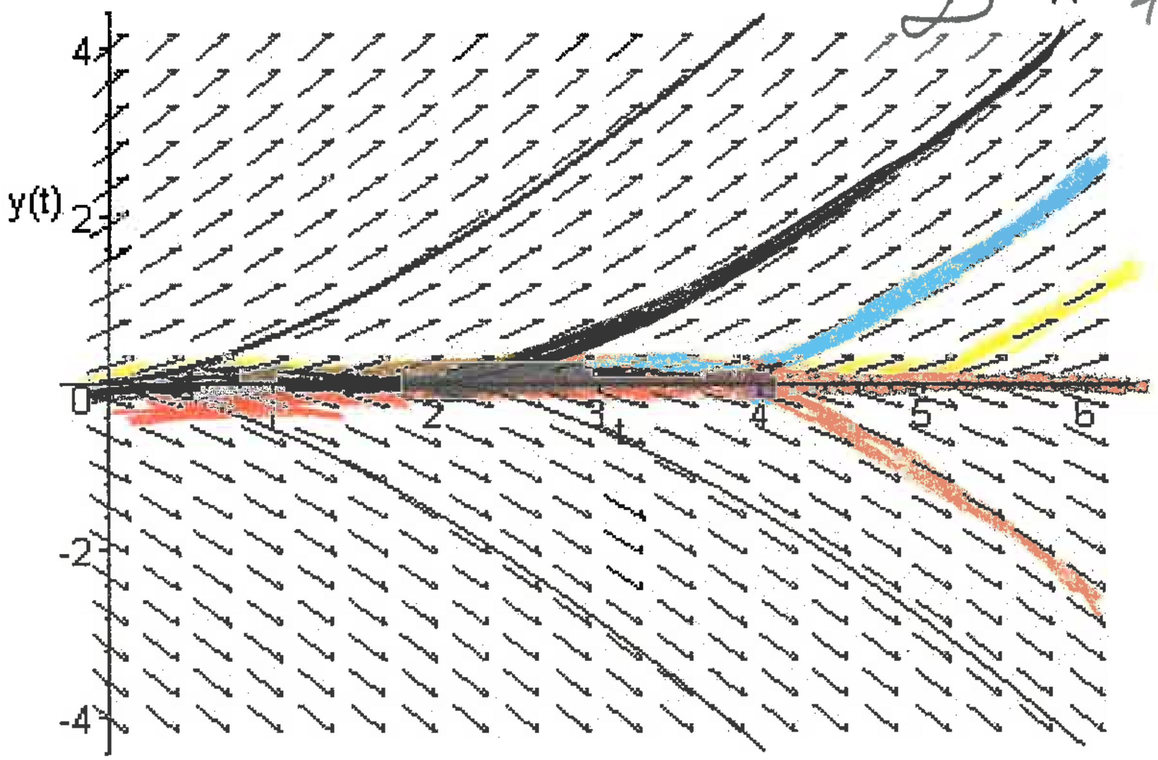
I) $y' = sin(t)$ | J) $y' = cos(t)$



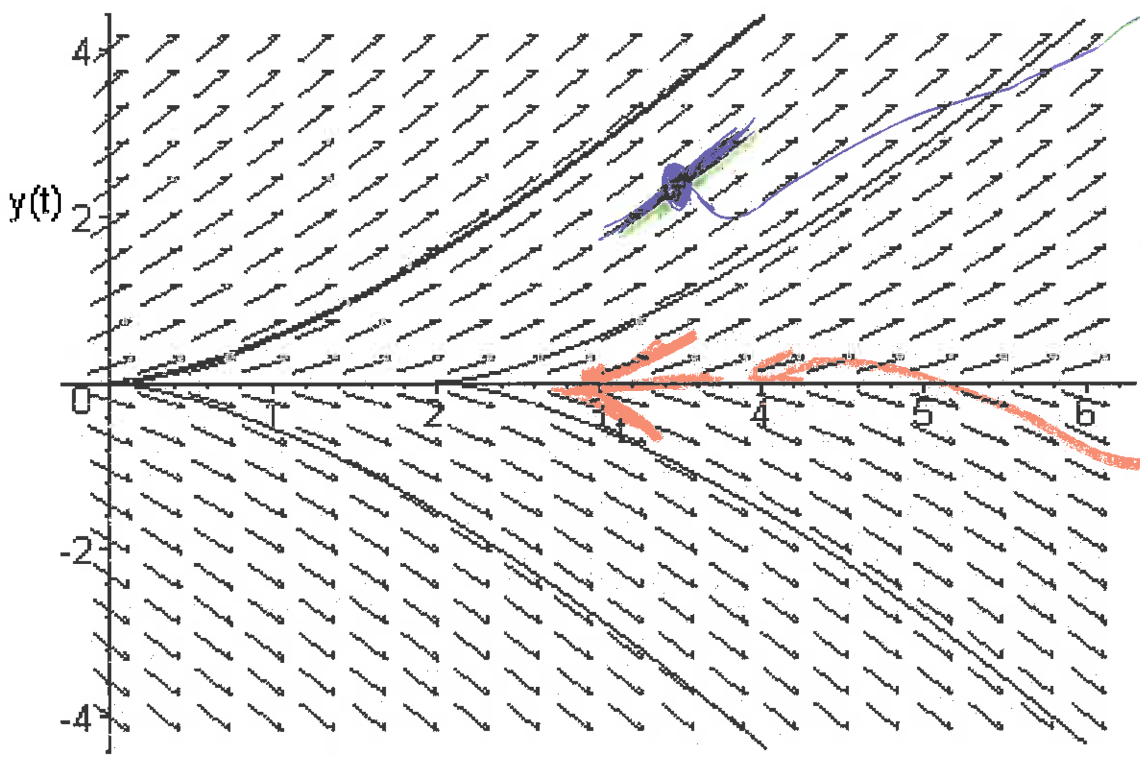
slope $0 = y - t \Rightarrow y = t$

$y' = y^{1/3}$, $y(4) = 0$

∞ # of sol'ns



y = 0 equilibrium soln



unique sol'n thru this initial value

not unique soln to IVP

Figure 2.4.1 from ***Elementary Differential Equations and Boundary Value Problems*, Eighth Edition** by William E. Boyce and Richard C. DiPrima

Calulus pre-requisites you must know.

Derivative = slope of tangent line = rate.

Integral = area between curve and x-axis (where area can be negative).

The Fundamental Theorem of Calculus: Suppose f continuous on $[a, b]$. $\Rightarrow$ Integral exists

1.) If $G(x) = \int_a^x f(t)dt$, then $G'(x) = f(x)$.

$$\text{I.e., } \frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x).$$

2.) $\int_a^b f(t)dt = F(b) - F(a)$ where F is any antiderivative of f, that is $F' = f$.

Suppose f is cont. on (a, b) and the point $t_0 \in (a, b)$, Solve IVP: $\frac{dy}{dt} = f(t)$, $y(t_0) = y_0$

$y' = f(t)$

If can separate variables

$$dy = f(t)dt$$

$$\int dy = \int f(t)dt$$

soln to IVP

$y = F(t) + C$ where F is any anti-derivative of F.

Initial Value Problem (IVP): $y(t_0) = y_0$

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$

Hence unique solution (if domain connected) to IVP:

$$y = F(t) + y_0 - F(t_0)$$

Note $y' = f(t)$ is NOT autonomous

Note: can use either section 2.1 method (integrating factor) or 2.2 method (separation of variables) to solve ex 2 and 3.

Ex 1: $t^2y' + 2ty = sin(t)$
(note, cannot use separation of variables).

$t^2y' + 2ty = sin(t)$

$(t^2y)' = sin(t)$ implies $\int (t^2y)'dt = \int sin(t)dt$

$(t^2y) = -cos(t) + C$ implies $y = -t^{-2}cos(t) + Ct^{-2}$

Gen ex: Solve $y' + p(x)y = g(x)$ LINEAR

Let $F(x)$ be an anti-derivative of $p(x)$. Thus $p(x) = F'(x)$

$$e^{F(x)}y' + [p(x)e^{F(x)}]y = g(x)e^{F(x)}$$

$$e^{F(x)}y' + [F'(x)e^{F(x)}]y = g(x)e^{F(x)}$$

$$[e^{F(x)}y]' = g(x)e^{F(x)}$$

$$e^{F(x)}y = \int g(x)e^{F(x)}dx$$

$\leftarrow$ exists if integral exists
including $F(x) = \int p(x)$ integral exists
p, g cont $\Rightarrow$ integral exists

$$y = e^{-F(x)} \int g(x)e^{F(x)}dx$$

LOOKING AT GENERAL CASE

CH 2: Solve $\frac{dy}{dt} = f(t, y)$

1.1: Direction Fields ***********************

*******Existence/Uniqueness of solution**********

Thm 2.4.2: Suppose the functions
$z = f(t, y)$ and $z = \frac{\partial f}{\partial y}(t, y)$ are cont. on $(a, b) \times (c, d)$
and the point $(t_0, y_0) \in (a, b) \times (c, d)$,
then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$
such that there exists a unique function $y = \phi(t)$
defined on $(t_0 - h, t_0 + h)$ that satisfies the following
initial value problem:

$$y' = f(t, y), \quad y(t_0) = y_0.$$

LOOKING AT LINEAR CASE

Thm 2.4.1: If p and g are continuous on (a, b) and the
point $t_0 \in (a, b)$, then there exists a unique function
$y = \phi(t)$ defined on (a, b) that satisfies the following
initial value problem:

$$\boxed{y' + p(t)y = g(t), \quad y(t_0) = y_0.}$$

LINEAR

But in general, $y' = f(t, y)$, solution may or may not
exist and solution may or may not be unique.

2.4 #27b. Solve Bernoulli's equation,

$$y' + p(t)y = g(t)y^n,$$

$$y^n \qquad y^n \qquad y^n$$

when $n \neq 0, 1$ by changing it

$$y^{-n}y' + p(t)y^{1-n} = g(t)$$

$$\|$$

when $n \neq 0, 1$ by changing it to a linear equation by substituting $v = y^{1-n}$ $\quad v \Rightarrow v' = (1-n)(y^{-n}y')$

Solve $ty' + 2t^{-2}y = 2t^{-2}y^5$

Section 2.5 Autonomous equations: $y' = f(y)$

Solve $\frac{dy}{dt} = f(y)$

If given either differential equation $y' = f(y)$

OR direction field:

Find equilibrium solutions and determine if stable, unstable, semi-stable.

Understand what the above means.