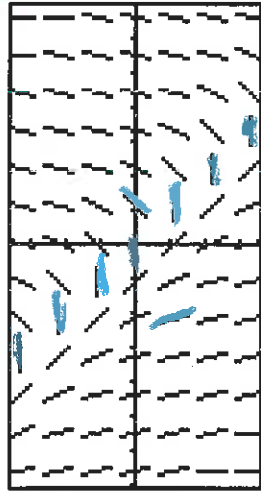
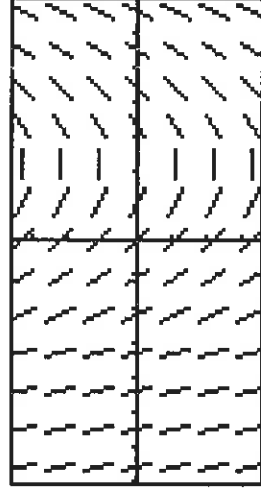


Match the slope fields with their differential equations.

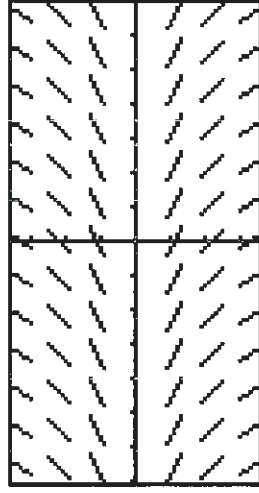
(A)



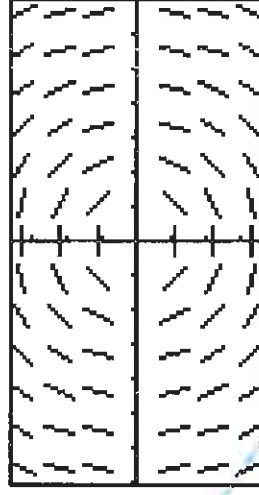
(B)



(C)



(D)



11. $\frac{dy}{dx} = 0.5x - 1$

12. $\frac{dy}{dx} = 0.5y$

13. $\frac{dy}{dx} = -\frac{x}{y}$

14. $\frac{dy}{dx} = x + y$

1, 2 = 2.2

<http://bes.wiley.com/bs-books/action/resource&bsId=2026&itemId=047143339X&resourceId=4140>

Ch 2.2: Separable Equations

In this section we examine a subclass of linear and nonlinear first order equations. Consider the first order equation

$$\frac{dy}{dx} = f(x, y)$$

We can rewrite this in the form

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

For example, let $M(x, y) = -f(x, y)$ and $N(x, y) = 1$. There may be other ways as well. In differential form,

$$M(x, y)dx + N(x, y)dy = 0$$

If M is a function of x only and N is a function of y only, then

$$M(x)dx + N(y)dy = 0$$

In this case, the equation is called separable.

Example 1: Solving a Separable Equation

Solve the following first order nonlinear equation:

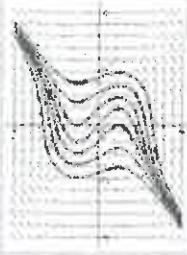
$$\frac{dy}{dx} = \frac{y^2 + 1}{y^2 - 1}$$

Separating variables, and using calculus, we obtain

$$\int \frac{(y^2 - 1)dy}{(y^2 - 1)(y^2 + 1)} = \int \frac{1}{(x^2 + 1)} dx$$

$$\frac{1}{3} y^3 - y = \frac{1}{3} x^3 + x + C$$

$$y^3 - 3y = x^3 + 3x + C$$



The equation above defines the solution implicitly. A graph showing the direction field and implicit plots of several integral curves for the differential equation is given above.

when can't solve for y

Example 2:

Implicit and Explicit Solutions (1 of 4)

Solve the following first order nonlinear equation:

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y - 1)}$$

Separating variables and using calculus, we obtain

$$2(y - 1)dy = \int (3x^2 + 4x + 2) dx$$

$$2 \int (y - 1)dy = \int (3x^2 + 4x + 2) dx$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + C$$

The equation above defines the solution implicitly. An explicit expression for the solution can be found in this case:

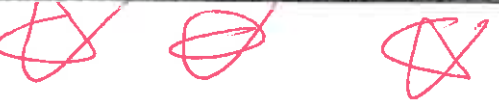
$$y^2 - 2y - (x^3 + 2x^2 + 2x + C) = 0 \Rightarrow y = \frac{2 \pm \sqrt{4 + 4(x^3 + 2x^2 + 2x + C)}}{2}$$

$$y = 1 \pm \sqrt{x^3 + 2x^2 + 2x + C}$$

quadratic formula

We can solve for y

quadratic formula



Example 2: Initial Value Problem (2 of 4)

Suppose we seek a solution satisfying $y(0) = -1$. Using the implicit expression of y , we obtain

$$y^2 - 2y = x^3 + 2x^2 + 2x + C$$

$$(-1)^2 - 2(-1) = C \Rightarrow C = 3$$

Thus the implicit equation defining y is

$$y^2 - 2y = x^3 + 2x^2 + 2x + 3$$

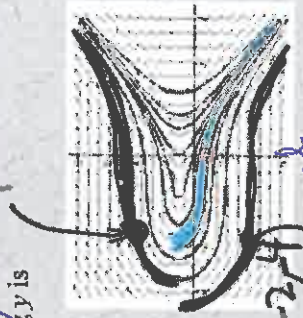
Using explicit expression of y ,

$$y = 1 \pm \sqrt{x^3 + 2x^2 + 2x + C}$$

$$-1 = 1 \pm \sqrt{C} \Rightarrow C = 4$$

It follows that

$$y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4}$$



$-1 = 1 + \sqrt{C} \Rightarrow$ no soln
 $-1 = 1 - \sqrt{C} \Rightarrow C = 4$

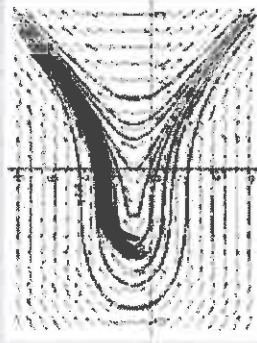
no soln



Example 2: Initial Condition $y(0) = 3$ (of 4)

Note that if initial condition is $y(0) = 3$, then we choose the positive sign, instead of negative sign, on square root term:

$$y = 1 + \sqrt{x^2 + 2x + 4}$$



where is valid
sdm valid

Example 2: Domain (4 of 4)

Thus the solutions to the initial value problem

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -1$$

are given by

$$y^2 - 2y = x^3 + 2x^2 + 2x + 3 \quad (\text{implicit})$$

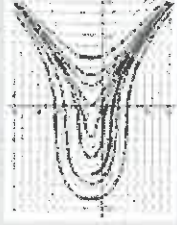
$$y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4} \quad (\text{explicit})$$

From explicit representation of y , it follows that

$$y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4} = 1 - \sqrt{(x+2)(x^2 + 2)}$$

and hence domain of y is $(-2, \infty)$. Note $x = -2$ yields $y = 1$, which makes denominator of dy/dx zero (vertical tangent).

Conversely, domain of y can be estimated by locating vertical tangents on graph (useful for implicitly defined solutions).



Example 3: Implicit Solution of Initial Value Problem (1 of 2)

Consider the following initial value problem:

$$y' = \frac{y \cos x}{1 + 3y^2}, \quad y(0) = 1$$

Separating variables and using calculus, we obtain

$$\frac{1 + 3y^2}{y} dy = \cos x dx$$

$$\int \left(\frac{1}{y} + 3y \right) dy = \int \cos x dx$$

$$\ln|y| + y^3 = \sin x + C$$

Using the initial condition, it follows that

$$\ln y + y^3 = \sin x + 1$$

Example 3: Graph of Solutions (2 of 2)

Thus

$$y' = \frac{y \cos x}{1 + 3y^2}, \quad y(0) = 1 \Rightarrow \ln y + y^3 = \sin x + 1$$

The graph of this solution (black), along with the graphs of the direction field and several integral curves (blue) for this differential equation, is given below.

