http://www.wolframalpha.com

slope field

Web Apps Examples Random

Assuming "slope field" refers to a computation | Use as referring to a mathematical definition instead

- vector field: {1, (ln(x) + y)/sqrt(1 ...
- variable 1: x
- lower limit 1: 0
- upper limit 1: 2
- variable 2: y
- lower limit 2: -2
- upper limit 2: 2

{1, (ln(x) + y)}/sqrt(1 + (ln(x) + y)^2)

(dx, dy)/length

(run, rise)

$y' = \ln x + y$

$\frac{dy}{dx} = \frac{\ln x + y}{1} = \frac{\text{rise}}{\text{run}}$

vector

(dx, dy)

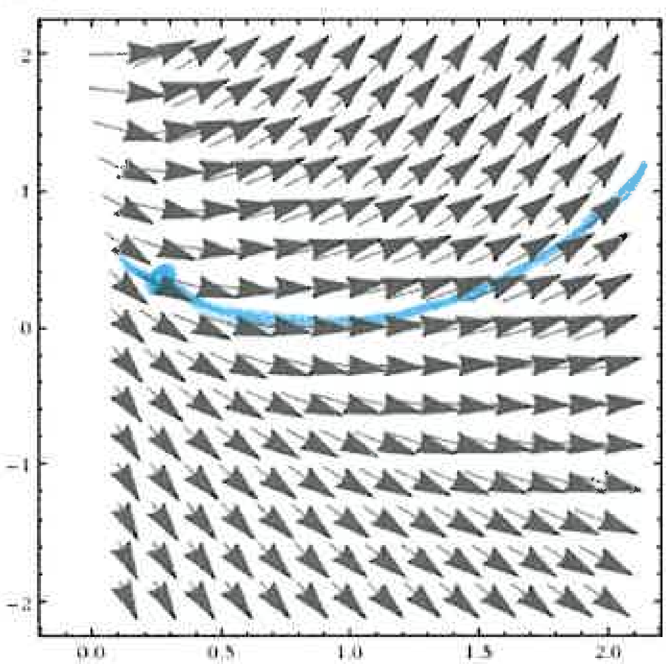
$\frac{(1, \ln x + y)}{\sqrt{1 + (\ln x + y)^2}}$

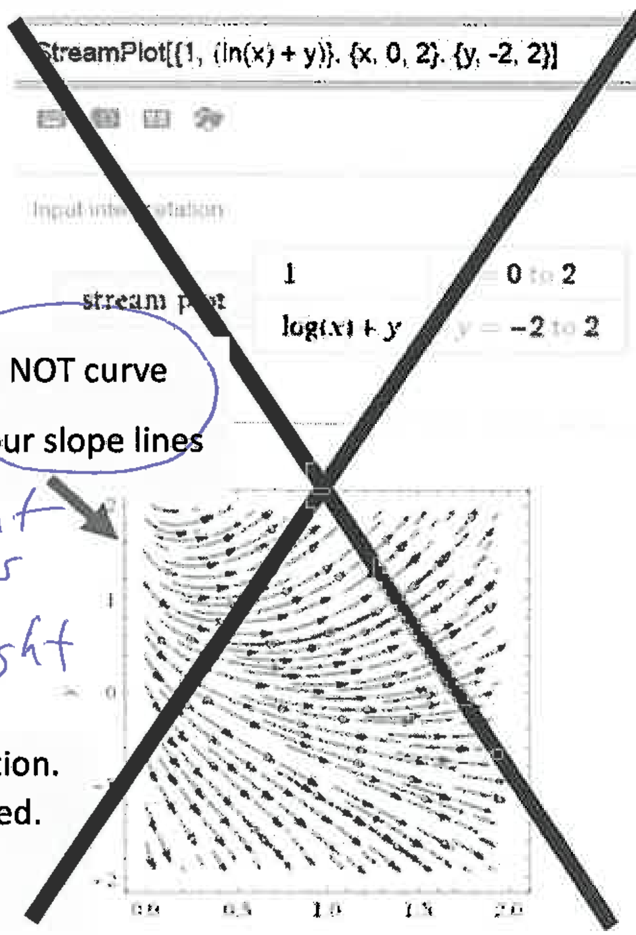
Input

$$\text{VectorPlot}\left[\frac{\{1, \log(x) + y\}}{\sqrt{1 + (\log(x) + y)^2}}, \{x, 0, 2\}, \{y, -2, 2\}\right]$$

log(x) is the natural logarithm

Result





Do NOT curve your slope lines

tangent lines are straight

Slope lines are small portions of lines tangent to a solution. Thus slope lines must be straight. They cannot be curved.

Arrows are optional

3.) Continuous compounding $\frac{dS}{dt} = rS + k$

$r \to 0$

where $S(t)$ = amount of money at time t,

r = interest rate,

k = constant deposit rate

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direction field = slope field = graph of $\frac{dv}{dt}$ in t, v-plane.

*** can use slope field to determine behavior of v including as $t \to \pm\infty$.

*** Equilibrium Solution = constant solution

A differential equation can have 0, 1, or multiple equilibrium solutions.

1.3:

ODE (ordinary differential equation): single independent variable

Ex: $\frac{dy}{dt} = ay + b$

PDE (partial differential equation): several independent variables

Ex: $\frac{\partial xy}{\partial x} = \frac{\partial xy}{\partial y}$

$2y' + 8 = y$ | $y \cdot y''' = 0$ | $y^{(99)} + y^2 = t$

first order | 3rd order | 99th order

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order of differential eq'n: order of highest derivative

example of order n: $y^{(n)} = f(t, y, ..., y^{(n-1)})$

Linear vs Non-linear

linear comb of $y, y', y'', ... y^{(n)}$

linear: $a_0(t)y^{(n)} + ... + a_n(t)y = g(t)$

linear w/ constant coeff Ch3

Determine if linear or non-linear:

2nd order linear

Ex: $ty'' - t^3y' - 3y = sin(t)$

linear 2nd order DE

Ex: $2y'' - 3y' - 3y^2 = 0$

NOT LINEAR

********Existence of a solution**************

********Uniqueness of solution***************

1.2: Solve $\frac{dy}{dt} = ay + b$ by separating variables:

Let $u = ay + b$ $\quad du = a\,dy$

$\frac{dy}{ay+b} = dt \quad \Rightarrow \quad \frac{1}{a}\int \frac{a\,dy}{ay+b} = \int dt \quad \Rightarrow \quad \frac{ln|ay+b|}{a} = t + C$

$ln|ay + b| = at + C \qquad$ implies $\qquad e^{ln|ay+b|} = e^{at+C}$

$|ay + b| = e^C e^{at} \qquad$ implies $\qquad ay + b = \pm(e^C e^{at})$

$ay = Ce^{at} - b \qquad$ implies $\qquad y = Ce^{at} - \frac{b}{a}$

STANDARD METHOD FOR SOLVING DE: Educated Guessing

Show that for some value of r, $y = e^{rt}$ is a soln to the 1^{rst} order linear homogeneous equation $2y' + 6y = 0$
constant coeff

To show something is a solution, plug it in:

$y = e^{rt}$ implies $y' = re^{rt}$. Plug into $2y' + 6y = 0$:

$2r\cancel{e^{rt}} + 6\cancel{e^{rt}} = 0$ implies $2r + 6 = 0$ implies $r = -3$

Thus $y = e^{-3t}$ is a solution to $2y' + 6y = 0$. $r = -3$

Show $y = Ce^{-3t}$ is a solution to $2y' + 6y = 0$.

$$\begin{aligned} 2y' + 6y = 2(Ce^{-3t})' + 6(Ce^{-3t}) &= 2C(e^{-3t})' + 6C(e^{-3t}) \\ &= C[2(e^{-3t})' + 6(e^{-3t})] = C(0) = 0. \end{aligned}$$

IVP If $y(0) = 4$, then $4 = Ce^{3(0)}$ implies $C = 4$.

Thus by existence and uniqueness thm, $y = 4e^{-3t}$ is the unique solution to IVP: $2y' + 6y = 0$, $y(0) = 4$.

CH 2: Solve $\frac{dy}{dt} = f(t, y)$

2.2: Separation of variables: $N(y)dy = P(t)dt$

2.1: First order linear eqn: $\frac{dy}{dt} + p(t)y = g(t)$

Ex 1: $t^2y' + 2ty = tsin(t)$

Ex 2: $y' = ay + b$

Ex 3: $y' + 3t^2y = t^2$, $y(0) = 0$

§1.2 = §2.2 separation of variables

http://bcs.wiley.com/he-bcs/Books?action=resource&bcsId=2026&itemId=047143339X&resourceId=4140

Ch 2.2: Separable Equations

- In this section we examine a subclass of linear and nonlinear first order equations. Consider the first order equation

$$\frac{dy}{dx} = f(x,y)$$

- We can rewrite this in the form

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

- For example, let $M(x,y) = -f(x,y)$ and $N(x,y) = 1$. There may be other ways as well. In differential form,

$$M(x,y)dx + N(x,y)dy = 0$$

- If M is a function of x only and N is a function of y only, then

$$M(x)dx + N(y)dy = 0$$

- In this case, the equation is called **separable**.

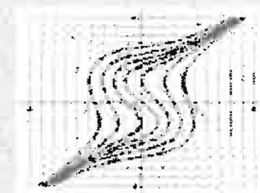
Example 1: Solving a Separable Equation

- Solve the following first order nonlinear equation:

$$\frac{dy}{dx} = \frac{x^2+1}{y^2-1}$$

- Separating variables, and using calculus, we obtain

$$\boxed{(y^2-1)dy} = \boxed{(x^2+1)dx}$$
$$\int (y^2-1)dy = \int (x^2+1)dx$$
$$\frac{1}{3}y^3 - y = \frac{1}{3}x^3 + x + C$$
$$y^3 - 3y = x^3 + 3x + C$$



- The equation above defines the solution y implicitly. A graph showing the direction field and implicit plots of several integral curves for the differential equation is given above.

can't solve for y so leave answer as implicit soln

Example 2:
Implicit and Explicit Solutions (1 of 4)

- Solve the following first order nonlinear equation:

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}$$

- Separating variables and using calculus, we obtain

$$2(y-1)dy = (3x^2+4x+2)dx$$
$$2\int (y-1)dy = \int (3x^2+4x+2)dx$$
$$y^2 - 2y = x^3 + 2x^2 + 2x + C$$

quadratic eqn

- The equation above defines the solution y implicitly. An explicit expression for the solution can be found in this case:

$$y^2 - 2y - (x^3+2x^2+2x+C) = 0 \Rightarrow y = \frac{2 \pm \sqrt{4+4(x^3+2x^2+2x+C)}}{2}$$
$$y = 1 \pm \sqrt{x^3+2x^2+2x+C}$$

quadratic formula

Solve for y

Example 2: Initial Value Problem (2 of 4)

- Suppose we seek a solution satisfying $y(0) = -1$. Using the implicit expression of y, we obtain

$$y^2 - 2y = x^3 + 2x^2 + 2x + C$$
$$(-1)^2 - 2(-1) = C \Rightarrow C = 3$$

- Thus the implicit equation defining y is

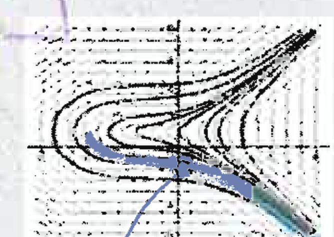
$$y^2 - 2y = x^3 + 2x^2 + 2x + 3$$

- Using explicit expression of y,

$$y = 1 \pm \sqrt{x^3+2x^2+2x+C}$$
$$-1 = 1 \pm \sqrt{C} \Rightarrow C = 4$$

- It follows that

$$y = 1 - \sqrt{x^3+2x^2+2x+4}$$



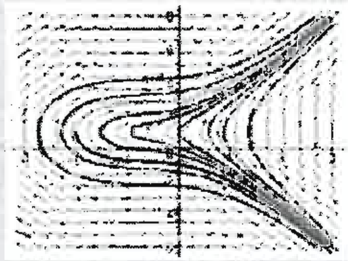
Choose − sign

(0, −1) initial value

Example 2: Initial Condition $y(0) = 3$ (3 of 4)

- Note that if initial condition is $y(0) = 3$, then we choose the positive sign, instead of negative sign, on square root term:

$$y = 1 + \sqrt{x^3 + 2x^2 + 2x + 4}$$



Example 2: Domain (4 of 4)

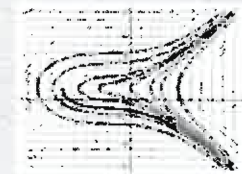
- Thus the solutions to the initial value problem

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -1$$

are given by

$$y^2 - 2y = x^3 + 2x^2 + 2x + 3 \quad \text{(implicit)}$$

$$y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4} \quad \text{(explicit)}$$



- From explicit representation of y, it follows that

$$y = 1 - \sqrt{x^2(x+2) + 2(x+2)} = 1 - \sqrt{(x+2)(x^2+2)}$$

and hence domain of y is $(-2, \infty)$. Note $x = -2$ yields $y = 1$, which makes denominator of dy/dx zero (vertical tangent).

- Conversely, domain of y can be estimated by locating vertical tangents on graph (useful for implicitly defined solutions).

Example 3: Implicit Solution of Initial Value Problem (1 of 2)

- Consider the following initial value problem:

$$y' = \frac{y\cos x}{1 + 3y^3}, \quad y(0) = 1$$

- Separating variables and using calculus, we obtain

$$\frac{1 + 3y^3}{y} dy = \cos x dx$$

$$\int \left(\frac{1}{y} + 3y^2\right) dy = \int \cos x dx$$

$$\ln|y| + y^3 = \sin x + C$$

- Using the initial condition, it follows that

$$\ln y + y^3 = \sin x + 1$$

Example 3: Graph of Solutions (2 of 2)

- Thus

$$y' = \frac{y\cos x}{1 + 3y^3}, \quad y(0) = 1 \Rightarrow \ln y + y^3 = \sin x + 1$$

- The graph of this solution (black), along with the graphs of the direction field and several integral curves (blue) for this differential equation, is given below.