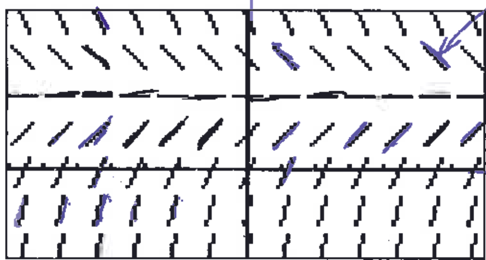
⇒ graph of small portions of tangent lines

Match the ~~slope~~ fields with their differential equations.

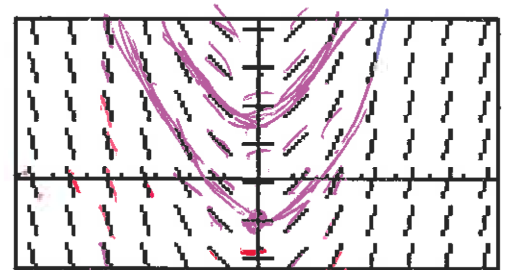
(A)



slope depends only on y

$y'(x) = f(y)$

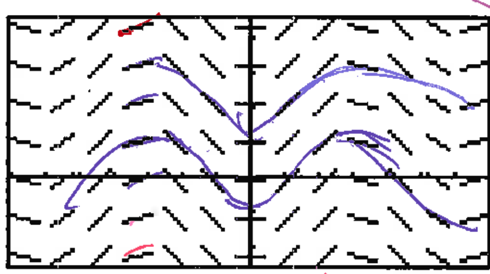
(B)



slope only depends on x

$y'(x) = f(x)$

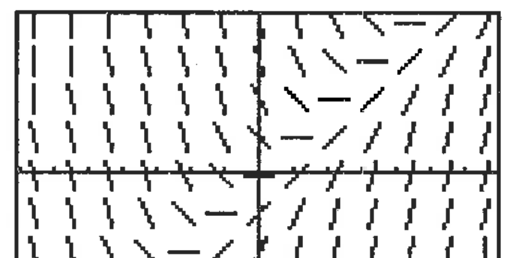
(C)



slope only depends on x

$y'(x) = g(x)$

(D)



slope depend on both x & y

$y' = f(x, y)$

7. $\frac{dy}{dx} = \sin x$ 8. $\frac{dy}{dx} = x - y$ 9. $\frac{dy}{dx} = 2 - y$ 10. $\frac{dy}{dx} = x$.

$\int dy = \int \sin x \, dx$

http://apcentral.collegeboard.com/apc/public/repository/ap08_calculus_slopefields_worksheet.pdf

$y = -\cos x + C$

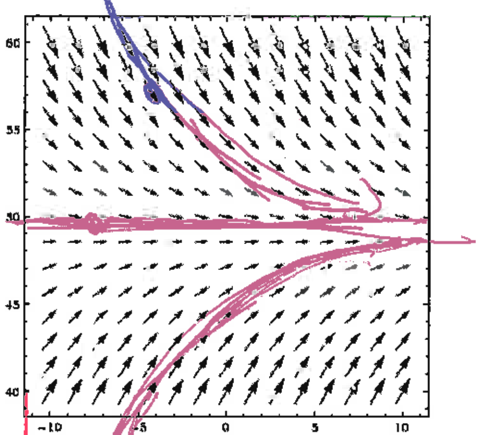
1.1: Examples of differentiable equation:

1.) $F = ma = m\frac{dv}{dt} = mg - \gamma v$

$v' = -\frac{\gamma}{m} v + g$

Eq soln

$y = 49$



$-\frac{\gamma}{m} < 0$

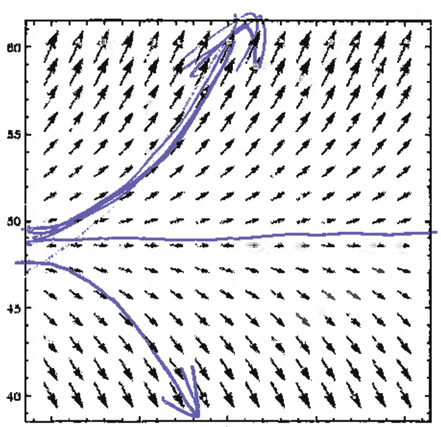
2.) Mouse population increases at a rate proportional to the current population:

More general model : $\frac{dp}{dt} = rp - k$

where $p(t)$ = mouse population at time t,

r = growth rate or rate constant,

k = predation rate = # mice killed per unit time.



$p' = rp - k$

$r > 0$

3.) Continuous compounding $\frac{dS}{dt} = rS + k$
where $S(t)$ = amount of money at time t,
r = interest rate,
k = constant deposit rate

$r > 0$

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direction field = slope field = graph of $\frac{dv}{dt}$ in t, v-plane.

*** can use slope field to determine behavior of v including as $t \to \pm\infty$.

*** Equilibrium Solution = constant solution

A differential equation can have 0, 1, or multiple equilibrium solutions.

1.3:

ODE (ordinary differential equation): single independent variable

Ex: $\frac{dy}{dt} = ay + b$

PDE (partial differential equation): several independent variables

Ex: $\frac{\partial xy}{\partial x} = \frac{\partial xy}{\partial y}$

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order of differential eq'n: order of highest derivative

example of order n: $y^{(n)} = f(t, y, ..., y^{(n-1)})$

Linear vs Non-linear

linear: $a_0(t)y^{(n)} + ... + a_n(t)y = g(t)$

Determine if linear or non-linear:

Ex: $ty'' - t^3y' - 3y = sin(t)$

Ex: $2y'' - 3y' - 3y^2 = 0$

********Existence of a solution**************

********Uniqueness of solution***************

1.2: Solve $\frac{dy}{dt} = ay + b$ by separating variables:

Let $u = ay + b$ $\quad du = a\,dy$

$\frac{dy}{ay+b} = dt \quad \Rightarrow \quad \frac{1}{a}\int \frac{a\,dy}{ay+b} = \int dt \quad \Rightarrow \quad \frac{ln|ay+b|}{a} = t + C$

$ln|ay + b| = at + C$ implies $e^{ln|ay+b|} = e^{at+C}$

$|ay + b| = e^C e^{at}$ implies $ay + b = \pm(e^C e^{at})$

$ay = Ce^{at} - b$ implies $y = Ce^{at} - \frac{b}{a}$