Thm 2.4.2: Suppose the functions
$z = f(t, y)$ and $z = \frac{\partial f}{\partial y}(t, y)$ are cont. on $(a, b) \times (c, d)$ and the point $(t_0, y_0) \in (a, b) \times (c, d)$,
then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$y' = f(t, y), \quad y(t_0) = y_0.$$

Thm 7.1.1: Suppose the functions
$z = F_i(t, x_1, ..., x_n)$ and $z = \frac{\partial F_i}{\partial x_j}(t, x_1, ..., x_n)$ are continuous for all i, j in a region $R =$
$\{(t, x_1, ..., x_n) \mid a < t < b, a_1 < x_1 < b_1, ..., a_n < x_n < b_n\}$,
and let the point $(t_0, x_1^0, ..., x_n^0) \in R$.
Then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that there exists a unique solution defined on $(t_0 - h, t_0 + h)$,

$$x_1 = \phi_1(t), \ ..., \ x_n = \phi_n(t)$$

that satisfies the following initial value problem:

$$\begin{aligned} x_1' &= F_1(t, x_1, ..., x_n) \\ x_2' &= F_2(t, x_1, ..., x_n) \\ &. \\ &. \\ &. \\ x_n' &= F_n(t, x_1, ..., x_n) \end{aligned}$$

$x_1(t_0) = x_1^0, x_2(t_0) = x_2^0, , ..., x_n(t_0) = x_n^0$

Theorem 4.1.1: If $p_i : (a,b) \to R$, $i = 1, ..., n$ and $g : (a,b) \to R$ are continuous and $a < t_0 < b$, then there exists a unique function $y = \phi(t)$, $\phi : (a,b) \to R$ that satisfies the initial value problem

$$y^{(n)} + p_1(t)y^{(n-1)} + ... + p_{n-1}(t)y' + p_n(t)y = g(t),$$
$$y(t_0) = y_0, \quad y'(t_0) = y_1, \quad y^{(n-1)}(t_0) = y_{n-1}$$

Thm 7.1.2: If p_{ij} and g_i are continuous on (a,b) and the point $t_0 \in (a,b)$, then there exists a unique solution $x_1 = \phi_1(t)$, ..., $x_n = \phi_n(t)$ defined on (a,b) that satisfies the following initial value problem:

$$\begin{pmatrix} x_1' \\ \cdot \\ \cdot \\ \cdot \\ x_n' \end{pmatrix} = \begin{pmatrix} p_{11}(t) & p_{12}(t) & ... & p_{1n}(t) \\ p_{21}(t) & p_{22}(t) & ... & p_{2n}(t) \\ & & \cdot & \\ & & \cdot & \\ & & \cdot & \\ p_{n1}(t) & p_{n2}(t) & ... & p_{nn}(t) \end{pmatrix} \begin{pmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix} + \begin{pmatrix} g_1(t) \\ \cdot \\ \cdot \\ \cdot \\ g_n(t) \end{pmatrix}$$

$x_1(t_0) = x_1^0, x_2(t_0) = x_2^0, , ..., x_n(t_0) = x_n^0$

Thm 7.4.1: If $\mathbf{f_k}(t)$ are solutions to $\mathbf{x}' = \mathbf{P}(t)\mathbf{x}$ where $P_{ij}(t) = p_{ij}(t)$, then the linear combination $\sum_{i=1}^{k} c_i \mathbf{f_k}(t)$ is also a solution for any constants c_i.

Thm 7.4.2: If $\mathbf{f_1}, ..., \mathbf{f_n}$ are linearly independent solutions to $\mathbf{x}' = \mathbf{P}(t)\mathbf{x}$ on (a,b), then if $\mathbf{x} = \mathbf{g}(t)$ is also a solution to this equation, then $\mathbf{g}(t) = \sum_{i=1}^{n} c_i \mathbf{f_k}(t)$ for some constants c_i