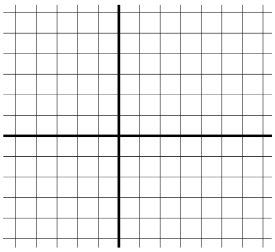
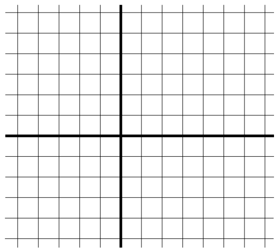
Example 1: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

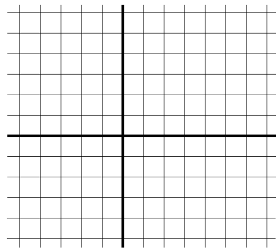
t, x_1-plane



t, x_2-plane

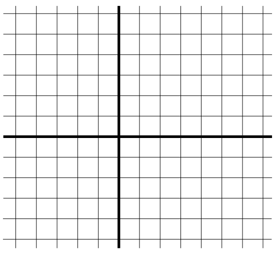


x_1, x_2-plane

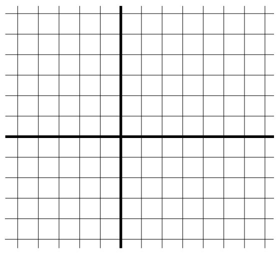


Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

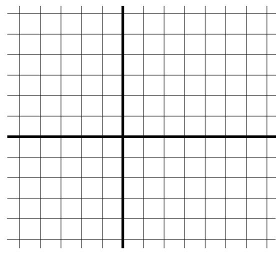
t, x_1-plane



t, x_2-plane



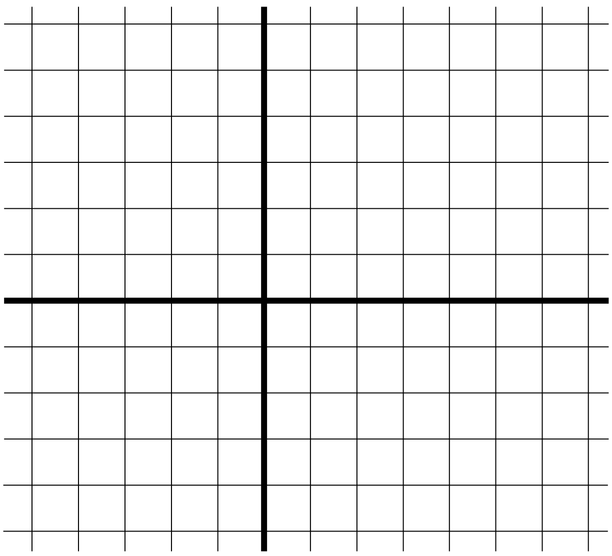
x_1, x_2-plane



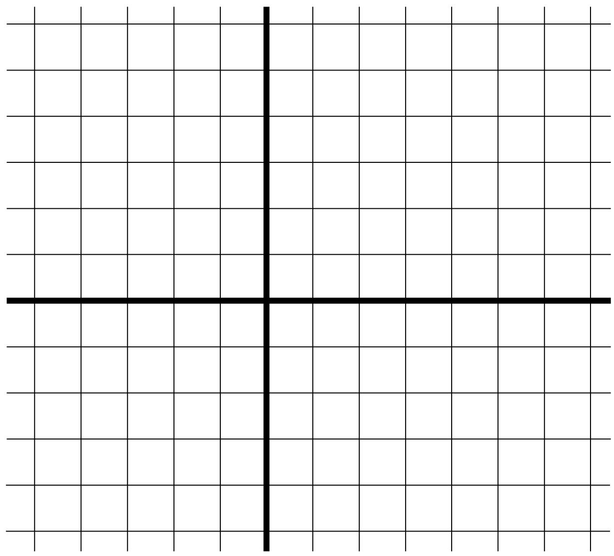
The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ________________

Plot several direction vectors where the slope is 0 and where slope is vertical.



Graph several trajectories.



Semi-generic ex: Given that the solution to to $\mathbf{x}' = A\mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

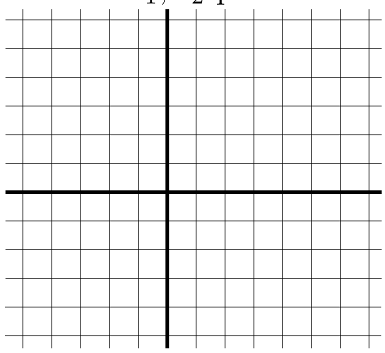
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ implies $c_1 = 1, c_2 = 0$. Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t}$

Hence $x_1 = -e^{r_1 t} < 0$ and $x_2 = 3e^{r_1 t} > 0$

and $\frac{x_2}{x_1} = \frac{3e^{r_1 t}}{-e^{r_1 t}} = \frac{3}{-1}$. Thus $x_2 = \frac{3}{-1} x_1$.

https://www.geogebra.org/3d (t, -e∧(-2t), 3*e∧(-2t))

x_1, x_2-plane



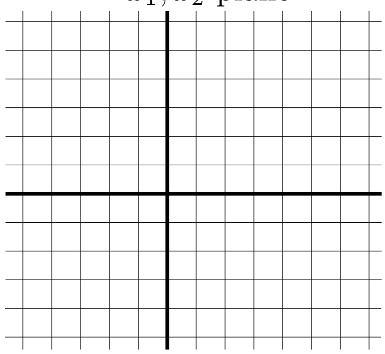
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ implies $c_1 = 2, c_2 = 0$. Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t}$

Hence $x_1 = -2e^{r_1 t} < 0$ and $x_2 = 6e^{r_1 t} > 0$

and $\frac{x_2}{x_1} = \frac{6e^{r_1 t}}{-2e^{r_1 t}} = \frac{3}{-1}$. Thus $x_2 = \frac{3}{-1} x_1$.

https://www.geogebra.org/3d (t, -2*e∧(-2t), 6*e∧(-2t))

x_1, x_2-plane



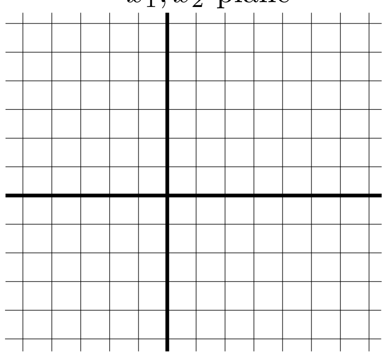
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ implies $c_1 = -1, c_2 = 0$. Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = - \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t}$

Hence $x_1 = e^{r_1 t} > 0$ and $x_2 = -3e^{r_1 t} < 0$

and $\frac{x_2}{x_1} = \frac{3e^{r_1 t}}{-e^{r_1 t}} = \frac{-3}{1}$. Thus $x_2 = \frac{-3}{1} x_1$.

https://www.geogebra.org/3d (t, e∧(-2t), -3*e∧(-2t))

x_1, x_2-plane



Semi-generic ex: Given that the solution to to $\mathbf{x}' = A\mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_2 t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ implies $c_1 = 0, c_2 = 1$.

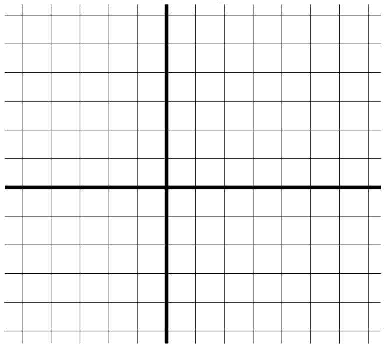
Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

Hence $x_1 = 0$ and $x_2 = e^{r_2 t} > 0$

and $\frac{x_2}{x_1} = \frac{1e^{r_2 t}}{0e^{r_2 t}} = \frac{1}{0}$. Thus $x_2 = \frac{1}{0} x_1$.

https://www.geogebra.org/3d (t, 0, e∧(5t))

x_1, x_2-plane



IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ implies $c_1 = 0, c_2 = -1$.

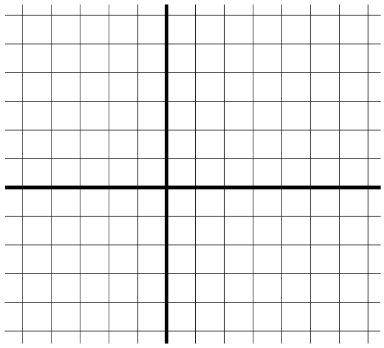
Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = - \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

Hence $x_1 = 0$ and $x_2 = -e^{r_2 t} < 0$

and $\frac{x_2}{x_1} = \frac{-1e^{r_2 t}}{0e^{r_2 t}} = \frac{-1}{0}$. Thus $x_2 = \frac{-1}{0} x_1$.

https://www.geogebra.org/3d (t, 0, -e∧(5t))

x_1, x_2-plane



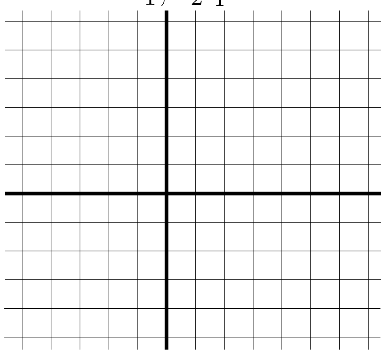
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ implies $c_1 = c_2 = 0$.

Thus IVP soln: $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Hence $x_1 = 0$ and $x_2 = 0$

https://www.geogebra.org/3d (t, 0, 0)

x_1, x_2-plane



Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

For large **positive** values of t which is larger: e^{-2t} or e^{5t}?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$.

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ______________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ______________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: e^{-2t} or e^{5t}?

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

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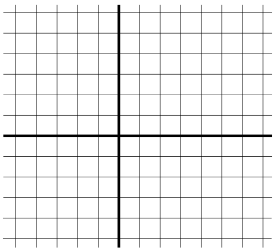
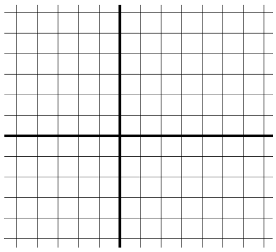
Example 2: Given that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

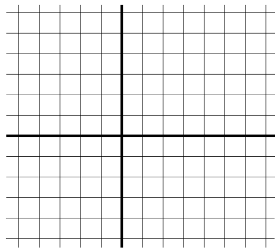
t, x_1-plane



t, x_2-plane

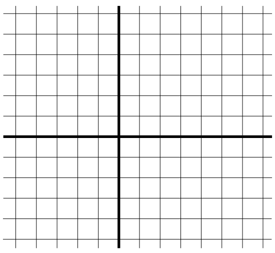


x_1, x_2-plane

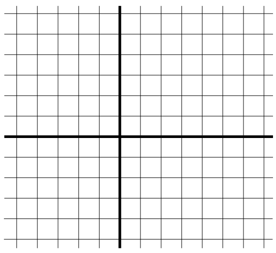


Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

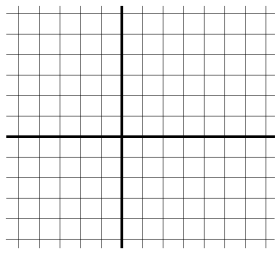
t, x_1-plane



t, x_2-plane



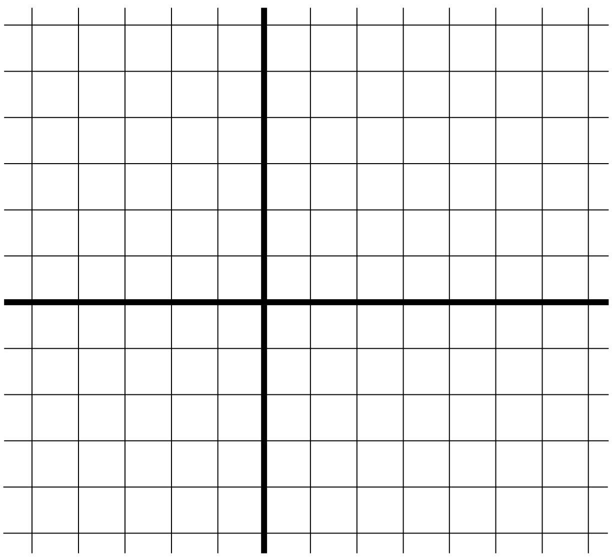
x_1, x_2-plane



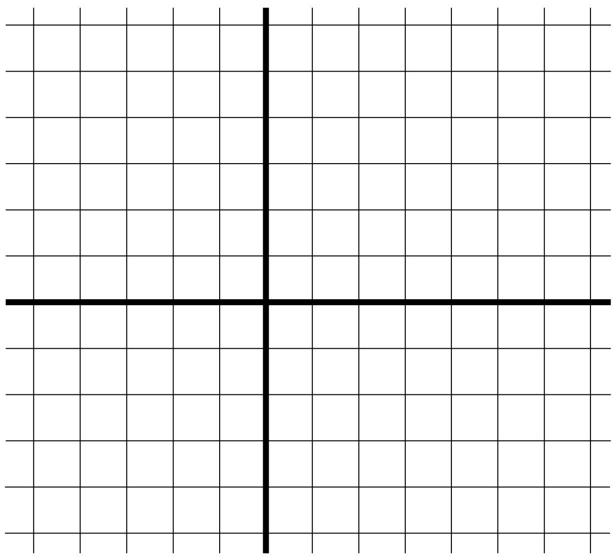
The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ________________

Plot several direction vectors where the slope is 0 and where slope is vertical.



Graph several trajectories.



Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

For large **positive** values of t which is larger: e^{-5t} or e^{-2t}?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t}$ or $c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$?

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ______________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ______________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: e^{-5t} or e^{-2t}?

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t}$ or $c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

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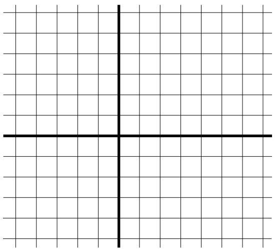
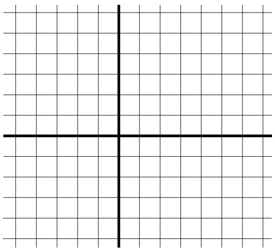
Example 3: Given that the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix}\mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

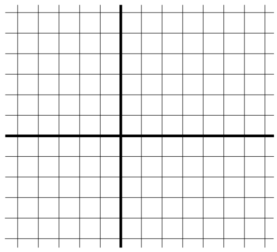
t, x_1-plane



t, x_2-plane

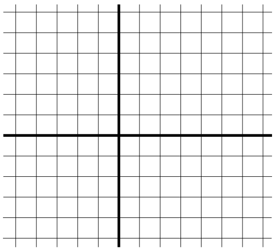


x_1, x_2-plane

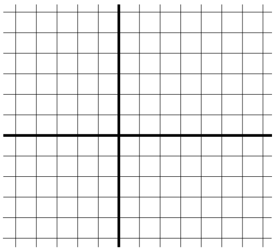


Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

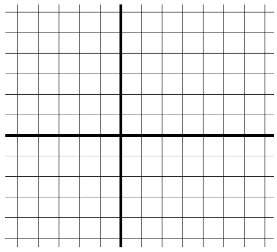
t, x_1-plane



t, x_2-plane



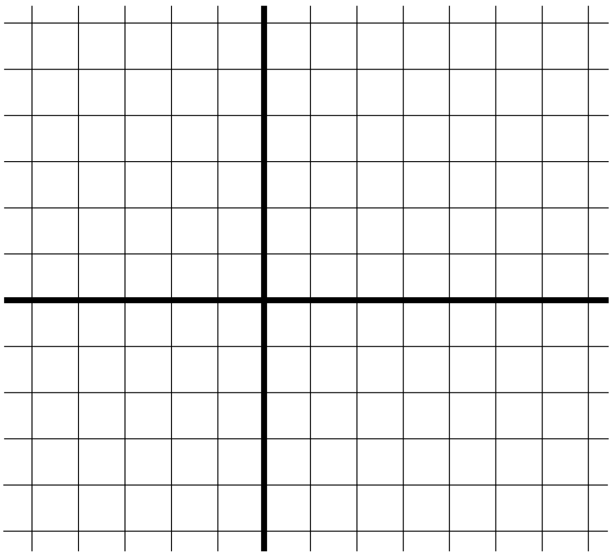
x_1, x_2-plane



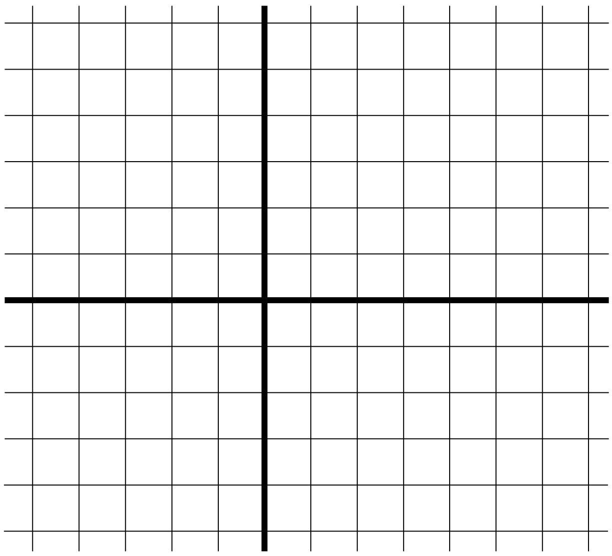
The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ________________

Plot several direction vectors where
the slope is 0 and where slope is vertical.



Graph several trajectories.



Answer the following questions for $A = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x} = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

For large **positive** values of t which is larger: e^{2t} or e^{5t}?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ______________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ______________. Note this case corresponds to where both $||c_1 \mathbf{v}|| e^{r_1 t}$ and $||c_2 \mathbf{w}|| e^{r_2 t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: e^{2t} or e^{5t}?

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

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* approaches the line $y = mx$ with slope $m =$ ______________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ______________. Note this case corresponds to where both $||c_1 \mathbf{v}|| e^{r_1 t}$ and $||c_2 \mathbf{w}|| e^{r_2 t}$ are large, but one is significantly larger than the other.

Find the **equilibrium solution(s)** for $\mathbf{x}' = A\mathbf{x}$ (Recall equilibrium solns are constant solns)

Recall a solution is an equilibrium solution iff $\mathbf{x}(t) = \mathbf{C}$ iff $\mathbf{x}'(t) = 0$

Setting $\mathbf{x}' = 0$, implies $\mathbf{0} = A\mathbf{x}$.

Thus $\mathbf{x} = \mathbf{C}$ is an equilibrium solution iff it is a solution to $\mathbf{0} = A\mathbf{x}$.

Case 1 (not emphasized/covered): $det(A) = 0$.

In this case, $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions. Note this case corresponds to the case when 0 is an eigenvalue of A since there are nonzero solutions to $A\mathbf{v} = 0\mathbf{v}$

Case 2: $det(A) \neq 0$.

Then $A\mathbf{x} = \mathbf{0}$ has a unique solution, $\mathbf{x} =$

Thus if $det(A) \neq 0$, $\mathbf{x} =$ is the only equilibrium solution of $\mathbf{x}' = A\mathbf{x}$

Slope fields:

* For complex eigenvalue case, one slope is needed.

* For real eigenvalue case, 0 and ∞ slopes can be helpful and can catch graphing errors, but your graph does **not** need to be that accurate.

For $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$

$\frac{dx_1}{dt} =$

$\frac{dx_2}{dt} =$

$\frac{dx_2}{dx_1} =$

Slope 0:

Slope ∞:

For $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$

$$\frac{dx_1}{dt} =$$

$$\frac{dx_2}{dt} =$$

$$\frac{dx_2}{dx_1} =$$

Slope 0:

Slope ∞:

For $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$

$$\frac{dx_1}{dt} =$$

$$\frac{dx_2}{dt} =$$

$$\frac{dx_2}{dx_1} =$$

Slope 0:

Slope ∞: