6.3: Step functions.

$$u_c(t) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$$

Graph $u_c(t)$:

Graph $g(t) = sin(t)$.

Graph $h(t) = u_\pi(t)sin(t)$.

Graph $f(t) = 2t + u_\pi(t)[sin(t) - 2t] = \begin{cases} & t < \pi \\ & t \geq \pi \end{cases}$

Example: $f(t) = \begin{cases} f_1, & \text{if } t < 4; \\ f_2, & \text{if } 4 \leq t < 5; \\ f_3, & \text{if } 5 \leq t < 10; \\ f_4, & \text{if } t \geq 10; \end{cases}$

Hence

$$f(t) = f_1(t) + u_4(t)[f_2(t) - f_1(t)] + u_5(t)[f_3(t) - f_2(t)] + u_{10}(t)[f_4(t) - f_3(t)]$$

Partial check:

If $t = 3$: $f(3) = f_1(3) + 0[f_2(3) - f_1(3)] + 0[f_3(3) - f_2(3)] + 0[f_4(3) - f_3(3)] = f_1(3)$

If $t = 9$: $f(9) = f_1(9) + 1[f_2(9) - f_1(9)] + 1[f_3(9) - f_2(9)] + 0[f_4(9) - f_3(9)] = f_3(9)$

Examples:

$f(t) = \begin{cases} 0 & 0 \leq t < 2 \\ t^2 & t \geq 2 \end{cases}$ implies $f(t) =$

$g(t) = \begin{cases} t^2 & 0 \leq t < 3 \\ 0 & t \geq 3 \end{cases}$ implies $g(t) =$

$h(t) = \begin{cases} t & 0 \leq t < 4 \\ ln(t) & t \geq 4 \end{cases}$ implies $h(t) =$

$$j(t) = \begin{cases} t & 0 \le t < 5 \\ 2 & 5 \le t \le 8 \\ e^t & t \ge 8 \end{cases} \quad \text{implies}$$

$j(t) =$

Formula 13: $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}F(s)$

Formula 13: $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t))$.

Let $g(t) = f(t+c)$. Then $g(t-c) = f(t-c+c) = f(t)$. Thus

$$\begin{aligned} \mathcal{L}(u_c(t)f(t)) = \mathcal{L}(u_c(t)g(t-c)) &= e^{-cs}\mathcal{L}(g(t)) \\ &= e^{-cs}\mathcal{L}(f(t+c)). \end{aligned}$$

or equivalently

$$\mathcal{L}(u_c(t)f(t)) = e^{-cs}\mathcal{L}(f(t+c)).$$

In other words, replacing $t-c$ with t is equivalent to replacing t with $t+c$

Find the LaPlace transform of the following:

a.) $\mathcal{L}(u_3(t)(t^2-2t+1)) =$ ______________

b.) $\mathcal{L}(u_4(t)(e^{-8t})) =$ ______________

c.) $\mathcal{L}(u_2(t)(t^2e^{3t})) =$ ______________

Find the LaPlace transform of

d.) $g(t) = \begin{cases} 0 & t < 3 \\ e^{t-3} & t \ge 3 \end{cases}$

e.) $f(t) = \begin{cases} 0 & t < 3 \\ 5 & 3 \leq t < 4 \\ t-5 & t \geq 4 \end{cases}$

Formula 13: $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t))$.

Let $F(s) = \mathcal{L}(f(t))$.

Then $\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}(\mathcal{L}(f(t))) = f(t)$.

Thus $\mathcal{L}^{-1}(e^{-cs}F(s))$

$$= \mathcal{L}^{-1}(e^{-cs}\mathcal{L}(f(t))) = u_c(t)f(t-c)$$

where $f(t) = \mathcal{L}^{-1}(F(s))$

Find the inverse LaPlace transform of the following:

a.) $\mathcal{L}^{-1}(e^{-8s}\frac{1}{s-3}) =$ ____________________

b.) $\mathcal{L}^{-1}(e^{-4s}\frac{1}{s^2-3}) =$ ____________________

c.) $\mathcal{L}^{-1}(e^{-s}\frac{5}{(s-3)^4}) =$ ____________________

d.) $\mathcal{L}^{-1}(\frac{e^{-s}}{4s}) =$ ____________________

e.) $\mathcal{L}^{-1}(e^{-s}) =$ ____________________

f.) $\mathcal{L}^{-1}(e^{-s}\frac{1}{(s-3)^2+4}) =$ ____________________

g.) $\mathcal{L}^{-1}(e^{-s}\frac{2s-5}{s^2+6s+13}) =$ ____________________