Solving polynomial equations:

$$\text{Example: } r^3 + r^2 + 3r + 10 = 0$$

Plug in $r = \pm 1, \pm 2, \pm 5, \pm 10$ to see if any of these are solns:

$$(\pm 1)^3 + (\pm 1)^2 + 3(\pm 1) + 10 \neq 0$$

$$(\pm 2)^3 + (\pm 2)^2 + 3(\pm 2) + 10 \ ? =? 0$$

$$(-2)^3 + (-2)^2 + 3(-2) + 10 = -8 + 4 - 6 + 10 = 0$$

Thus $(r - (-2))$ is a factor of $r^3 + r^2 + 3r + 10$

Hence $r^3 + r^2 + 3r + 10 = (r + 2)(r^2 + \underline{\quad\quad} r + 5)$

To find the coefficient of r in the above, you can do so by (1) long division, (2) inspection, (3) using variable x

$r^3 + r^2 + 3r + 10 = (r + 2)(r^2 + \underline{\ x\ } r + 5)$

$$(r + 2)(r^2 + \underline{\ x\ } r + 5) = r^3 + (2 + x)r^2 + (2x + 5)r + 10$$

$$r^3 \quad + \quad r^2 \quad + \quad 3r \quad + \quad 10$$

$$\text{Thus } 2 + x = 1 \text{ and hence } x = -1$$

$$\text{Check: } 2x + 5 = 2(-1) + 5 = 3$$

Hence $r^3 + r^2 + 3r + 10 = (r + 2)(r^2 - r + 5) = 0$

Thus $r = -2, \frac{1 \pm \sqrt{1-20}}{2}$. Thus $r = -2, \frac{1 \pm i\sqrt{19}}{2}$.

In special cases, you can use the unit circle.

Ex: $r^4 + 1 = 0$ implies

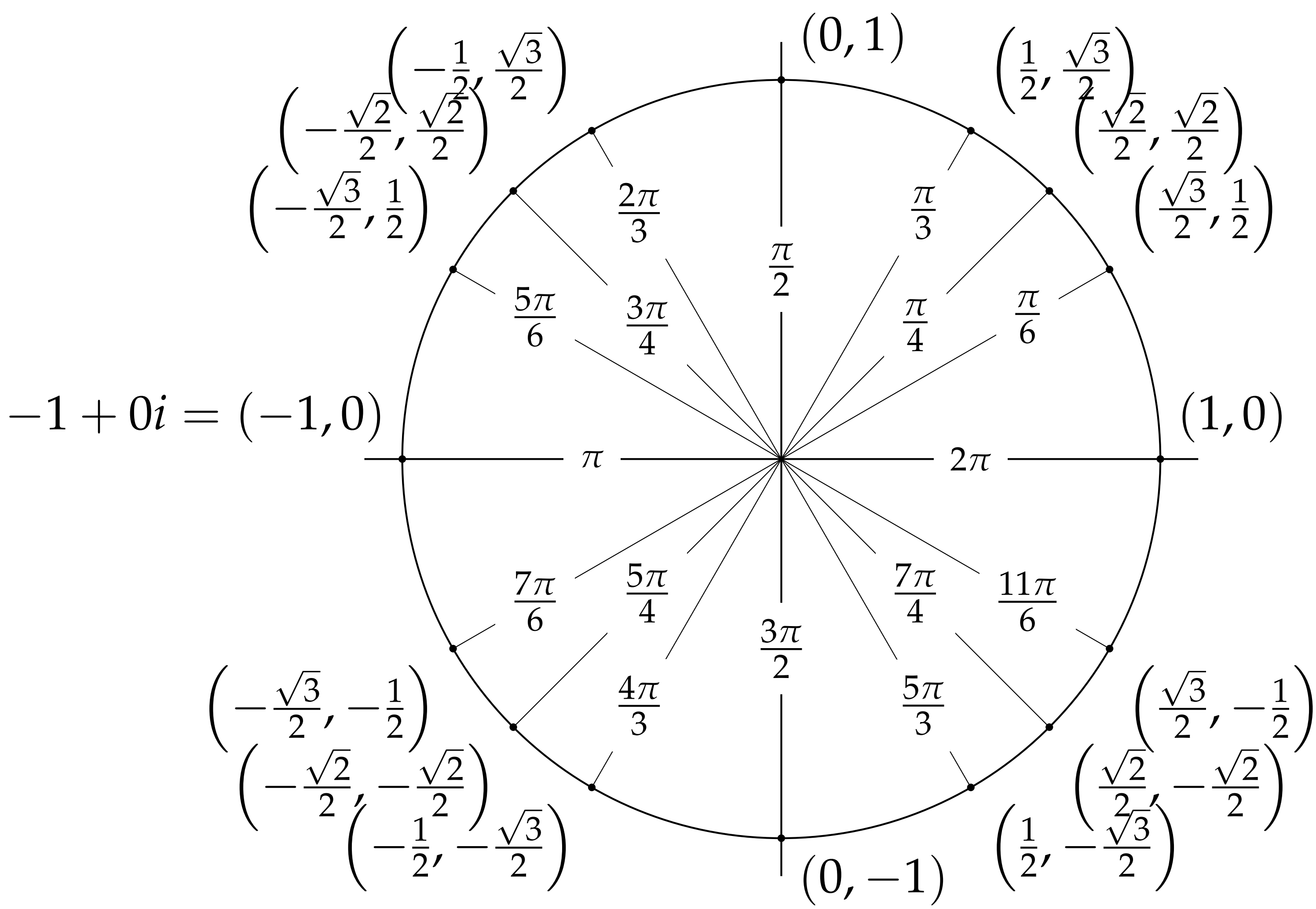
$$r = (-1)^{\frac{1}{4}} = (-1 + 0i)^{\frac{1}{4}} = (e^{i\pi})^{\frac{1}{4}} = (e^{i(\pi+2\pi k)})^{\frac{1}{4}}$$

$$k = 0: \qquad e^{\frac{i\pi}{4}} = cos(\tfrac{\pi}{4}) + isin(\tfrac{\pi}{4}) = \tfrac{\sqrt{2}}{2} + i\tfrac{\sqrt{2}}{2}$$

$$k = 1: \qquad e^{\frac{3i\pi}{4}} = cos(\tfrac{3\pi}{4}) + isin(\tfrac{3\pi}{4}) = -\tfrac{\sqrt{2}}{2} + i\tfrac{\sqrt{2}}{2}$$

$$k = 2: \qquad e^{\frac{5i\pi}{4}} = cos(\tfrac{5\pi}{4}) + isin(\tfrac{5\pi}{4}) = -\tfrac{\sqrt{2}}{2} - i\tfrac{\sqrt{2}}{2}$$

$$k = 3: \qquad e^{\frac{7i\pi}{4}} = cos(\tfrac{7\pi}{4}) + isin(\tfrac{7\pi}{4}) = \tfrac{\sqrt{2}}{2} - i\tfrac{\sqrt{2}}{2}$$



$sin(0) = \frac{\sqrt{0}}{2} = 0 \qquad cos(0) = 1$

$sin(\frac{\pi}{6}) = \frac{\sqrt{1}}{2} = \frac{1}{2} \qquad cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} \qquad cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} \qquad cos(\frac{\pi}{3}) = \frac{1}{2}$

$sin(\frac{\pi}{2}) = \frac{\sqrt{4}}{2} = 1 \qquad cos(\frac{\pi}{2}) = 0$

Example: Solve $y^{(iv)} + y = 0$

$y = e^{rt}$ implies $r^4 + 1 = 0$ and thus

$$r = \frac{\sqrt{2}}{2} \pm i\frac{\sqrt{2}}{2} \text{ and } r = -\frac{\sqrt{2}}{2} \pm i\frac{\sqrt{2}}{2}$$

Thus general homogeneous solution is

$$y = c_1 e^{\frac{\sqrt{2}}{2}t} cos(\frac{\sqrt{2}}{2}t) + c_2 e^{\frac{\sqrt{2}}{2}t} sin(\frac{\sqrt{2}}{2}t)$$
$$+ c_3 e^{-\frac{\sqrt{2}}{2}t} cos(\frac{\sqrt{2}}{2}t) + c_4 e^{-\frac{\sqrt{2}}{2}t} sin(\frac{\sqrt{2}}{2}t)$$