3.5: Solving non-homogeneous linear DE using the undetermined coefficients method

1.) Step 1: Solve homogeneous version of DE.

2.) Step 2: Guess a non-homogeneous solution with undetermined coefficients. Plug into the non-homogeneous linear DE to solve for the undetermined coefficients.

3.) Combing general homogeneous solution with a non-homogeneous solution.

Starting guess:

If $ay'' + by' + cy = ke^{pt}$, guess $y = Ae^{pt}$

If $ay'' + by' + cy = ksin(pt) + jcos(pt)$, guess $y = Asin(pt) + Bcos(pt)$

If $ay'' + by' + cy =$ degree n polynomial,
guess $y =$ a degree n polynomial including all terms (with undetermined coefficients) including constant term.

If $ay'' + by' + cy =$ a sum, guess a sum (but usually solve separately).

If $ay'' + by' + cy =$ a product, guess a product.

Sometimes the above can be simplified:

If a term does not show up when you take the derivatives of y, you may be able to omit that term. E.g, $y'' + w^2y = sin(pt)$ where $p \neq w$, then $y = Asin(pt)$ is a simpler guess that works.

If the above does not work

Try multiplying non-simplified guess by t.

Example: If guess is a homogeneous solution, then that will not be a non-homogeneous solution. Thus must guess something else. Multiplying non-simplified guess by t until no longer homogeneous works.

Example: If y term missing, and $g(t) =$ degree n polynomial, then will need to multiply by t so that when you plug in guess, you will have a degree n polynomial on both sides of equal sign.

Note: you are multiplying the **non-simplified guess** by t. When you take derivatives of y, you must use the **product** rule. Thus extra terms appear when you take the derivative and you will need the non-simplified guess to cancel out these terms.