Solve: $y'' + y = 0$, $y(0) = -1$, $y'(0) = -3$

$r^2 + 1 = 0$ implies $r^2 = -1$. Thus $r = \pm i$.

RECOMMENDED Method:

Since $r = 0 \pm 1i$, $y = k_1 cos(t) + k_2 sin(t)$

Then $y' = -k_1 sin(t) + k_2 cos(t)$

$y(0) = -1$: $-1 = k_1 cos(0) + k_2 sin(0)$ implies $-1 = k_1$

$y'(0) = -3$: $-3 = -k_1 sin(0) + k_2 cos(0)$ implies $-3 = k_2$

Thus IVP solution: $y = -cos(t) - 3sin(t)$

NOT RECOMMENDED: work with $y = c_1 e^{it} + c_2 e^{-it}$

$y' = ic_1 e^{it} - ic_2 e^{-it}$

$y(0) = -1$: $-1 = c_1 e^0 + c_2 e^0$ implies $-1 = c_1 + c_2$.

$y'(0) = -3$: $-3 = ic_1 e^0 - ic_2 e^0$ implies $-3 = ic_1 - ic_2$.

$-1i = ic_1 + ic_2$.

$-3 = ic_1 - ic_2$.

$2ic_1 = -3 - i$ implies $c_1 = \frac{-3i - i^2}{-2} = \frac{3i-1}{2}$

$2ic_2 = 3 - i$ implies $c_2 = \frac{3i - i^2}{-2} = \frac{-3i-1}{2}$

Euler's formula: $e^{ix} = cos(x) + isin(x)$

$y = (\frac{3i-1}{2})e^{it} + (\frac{-3i-1}{2})e^{-it} = (\frac{3i-1}{2})[cos(t) + isin(t)] + (\frac{-3i-1}{2})[cos(-t) + isin(-t)]$

$= (\frac{3i-1}{2})[cos(t) + isin(t)] + (\frac{-3i-1}{2})[cos(t) - isin(t)]$

$= (\frac{3i}{2})cos(t) + (\frac{3i}{2})isin(t) + (\frac{-1}{2})cos(t) + (\frac{-1}{2})isin(t) + (\frac{-3i}{2})cos(t) - (\frac{-3i}{2})isin(t) + (\frac{-1}{2})cos(t) - (\frac{-1}{2})isin(t)$

$= (\frac{3i}{2})isin(t) + (\frac{-1}{2})cos(t) + (\frac{3i}{2})isin(t) + (\frac{-1}{2})cos(t)$

$= -(\frac{3}{2})sin(t) - (\frac{1}{2})cos(t) - (\frac{3}{2})sin(t) - (\frac{1}{2})cos(t)$

$= -3sin(t) - 1cos(t)$