Calulus pre-requisites you must know.

Derivative = slope of tangent line = rate.

Integral = area between curve and x-axis (where area can be negative).

The Fundamental Theorem of Calculus: Suppose $f$ continuous on $[a, b]$.

1.) If $G(x) = \int_a^x f(t)dt$, then $G'(x) = f(x)$.

$$\text{I.e., } \frac{d}{dx}[\int_a^x f(t)dt] = f(x).$$

2.) $\int_a^b f(t)dt = F(b) - F(a)$ where $F$ is any antiderivative of $f$, that is $F' = f$.

Suppose $f$ is cont. on $(a, b)$ and the point $t_0 \in (a, b)$,
Solve IVP: $\frac{dy}{dt} = f(t)$, $y(t_0) = y_0$

$$dy = f(t)dt$$

$$\int dy = \int f(t)dt$$

$y = F(t) + C$ where $F$ is any anti-derivative of $F$.

Initial Value Problem (IVP): $y(t_0) = y_0$

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$

Hence unique solution (if domain connected) to IVP:

$$y = F(t) + y_0 - F(t_0)$$

CH 2: Solve $\frac{dy}{dt} = f(t, y)$

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***1.1: Direction Fields *************************

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*******Existence/Uniqueness of solution**********

Thm 2.4.2: Suppose the functions
$z = f(t, y)$ and $z = \frac{\partial f}{\partial y}(t, y)$ are cont. on $(a, b) \times (c, d)$
and the point $(t_0, y_0) \in (a, b) \times (c, d)$,
then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$y' = f(t, y), \quad y(t_0) = y_0.$$

Thm 2.4.1: If $p$ and $g$ are continuous on $(a, b)$ and the point $t_0 \in (a, b)$, then there exists a unique function $y = \phi(t)$ defined on $(a, b)$ that satisfies the following initial value problem:

$$y' + p(t)y = g(t), \quad y(t_0) = y_0.$$

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But in general, $y' = f(t, y)$, solution may or may not exist and solution may or may not be unique.

Ex 1: $y' = y' + 1$

Ex 2: $(y')^2 = -1$

IVP ex 3: $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$

$\int \frac{dy}{y} = \int (1 + \frac{1}{x})dx \quad$ implies $\quad ln|y| = x + ln|x| + C$

$|y| = e^{x + ln|x| + C} = e^x e^{ln|x|} e^C = C|x|e^x$

$y = \pm Cxe^x$ implies $y = Cxe^x$ or ...

$y(0) = 1: \quad 1 = C(0)e^0 = 0$ implies

IVP $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$ has no solution.

http://www.wolframalpha.com
slope field: $(1, y(1+1/x))/sqrt(1+y\wedge 2(1+1/x)\wedge 2)$

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**Ex Non-unique**: $y' = y^{\frac{1}{3}}$

$y = 0$ is a solution to $y' = y^{\frac{1}{3}}$ since $y' = 0 = 0^{\frac{1}{3}} = y^{\frac{1}{3}}$

Suppose $y \neq 0$. Then $\frac{dy}{dx} = y^{\frac{1}{3}}$ implies $y^{-\frac{1}{3}} dy = dx$

$\int y^{-\frac{1}{3}} dy = \int dx$ implies $\frac{3}{2} y^{\frac{2}{3}} = x + C$

$y^{\frac{2}{3}} = \frac{2}{3}x + C$ implies $y = \pm\sqrt{(\frac{2}{3}x + C)^3}$

Suppose $y(3) = 0$. Then $0 = \sqrt{(2 + C)^3}$ implies $C = -2$.

Thus initial value problem, $y' = y^{\frac{1}{3}}$, $y(3) = 0$, has 3 sol'ns:

$$y = 0, \quad y = \sqrt{(\frac{2}{3}x - 2)^3}, \quad y = -\sqrt{(\frac{2}{3}x - 2)^3}$$

2.4 #27b. Solve Bernoulli's equation,

$$y' + p(t)y = g(t)y^n,$$

when $n \neq 0, 1$ by changing it

$$y^{-n}y' + p(t)y^{1-n} = g(t)$$

when $n \neq 0, 1$ by changing it to a linear equation by substituting $v = y^{1-n}$

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Solve $ty' + 2t^{-2}y = 2t^{-2}y^5$

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Section 2.5: Solve $\frac{dy}{dt} = f(y)$

If given either differential equation $y' = f(y)$ OR direction field:

Find equilibrium solutions and determine if stable, unstable, semi-stable.

Understand what the above means.