Midterm I. (22C:231, Sample midterm test)  
Open Book and Notes, total points = 100

1. **Greedy Algorithms** (25)  
   Describe a greedy algorithm of making change for \( n \) cents using the fewest number of coins, where coins are quarters, dimes, nickels, pennies, and two-cent coins. Prove that your algorithm yields an optimal solution. For example, 6 cents can be paid using either one nickel and one cent, or three two-cent coins. The former is better because it uses two coins.

2. **Dynamic Programming** (25)  
   Suppose the supervisor relation in a company can be modeled by a tree rooted at the president, where each node of the tree represents a unique employee. The president wants to hold a party where both an employee and his/her immediate supervisor cannot attend at the same time. Please design an efficient algorithm that uses the supervisor relation tree and produces the maximal number of employees who can attend the party. Suppose there are \( n \) employees, please analyze the time complexity of your algorithm in terms of \( n \). For a given tree node \( x \), name\((x)\) is the employee’s name, supervisees\((x)\) is a list of tree nodes representing the employees supervised by \( x \).

3. **Dynamic Programming** (25)  
   Suppose that I am a freelance journalist who specializes in covering events at New York and San Francisco. In each of the next \( n \) months, I have a choice of covering either New York or San Francisco. If I am in New York in the \( i \)th month, I will get paid \( N_i \) dollars. If I am in San Francisco in the \( i \)th month, I will get paid \( S_i \) dollars. I do not having moving costs – if I decide to be in New York in month 5 and in San Francisco in month 6, my flight ticket will be reimbursed (the payments are separate from this). However, each flight I take will earn me \( F \) dollars worth of frequent flyer miles. Assume that I am in New York currently, so I will earn \( F \) frequent flyer dollars if I choose to be in San Francisco in month 1.

   The input consists of the sequence \( N_1, N_2, \ldots, N_n \), and the sequence \( S_1, S_2, \ldots, S_n \), and the number \( F \), and the goal is to find a plan that maximizes my earnings, which is the total payment made to me plus the total frequent flyer dollars I earn.

   Thus for example if \( n = 4 \), the plan NY NY SF NY results in an earnings of \( N_1 + N_2 + S_3 + N_4 + 2F \).

   Give a polynomial time algorithm to solve this problem. It is sufficient to return the earnings of the optimal plan, and not the optimal plan itself.

4. **Flow Network** (25)  
   Let \( G = (X, Y, E) \) be a bipartite graph, where \( X = \{x_1, x_2, x_3, x_4, x_5\} \), \( Y = \{y_1, y_2, y_3, y_4\} \), and \( E = \{(x_1, y_1), (x_2, y_1), (x_2, y_3), (x_3, y_2), (x_3, y_3), (x_3, y_4), (x_4, y_3), (x_5, y_3)\} \). You are asked to find a maximum matching of \( G \) using the max-flow algorithm.

   (a) (5) Please draw the corresponding flow network and label the capacity on each edge.  
   (b) (10) Please list all the augmenting paths that you used to find a maximum flow in the corresponding flow network.  
   (c) (10) Please list a minimum cut of the corresponding flow network.