7.9 Airline Scheduling

Given an airline schedule, and starting locations for the planes, is it possible to use a fixed set of planes to satisfy the schedule.

Schedule
- A set of segments: departure, arrival pairs (cities and times)

Approach
- Construct a circulation problem where paths of flow give segments flown by each plane

Example
Seattle -> San Francisco, 9:00 - 11:00
Seattle -> Denver, 8:00 - 11:00
San Francisco -> Los Angeles, 13:00 - 14:00
Salt Lake City -> Los Angeles, 15:00-17:00
San Diego -> Seattle, 17:30-> 20:00
Los Angeles -> Seattle, 18:00->20:00

Flight times:
- Denver -> Salt Lake City, 2 hours
- Los Angeles -> San Diego, 1 hour

Can this schedule be fulfilled with two planes, starting from Seattle?
Compatible segments

Segments S₁ and S₂ are compatible if the same plane can be used on S₁ and S₂.

- End of S₁ equals start of S₂, and enough time for turn around between arrival and departure times.
- End of S₁ is different from S₂, but there is enough time to fly between cities.

Graph representation

Each segment, Sᵢ, is represented as a pair of vertices (dᵢ, aᵢ) for departure and arrival, with an edge between them.

Add an edge between aᵢ and dⱼ if Sⱼ is reachable from Sᵢ.

Setting up a flow problem

Given a set S = {s₁, s₂, ... , sₙ} of flight segments, where sᵢ = (dᵢ, aᵢ), dᵢ = departing city and time, and aᵢ = arriving city and time, the reachability relation among segments.

We construct a flow network with lower bounds G = (V, E, l, c), where

- V = {dᵢ, aᵢ | sᵢ = (dᵢ, aᵢ) in S} U {s, t}
- E = S U { (aᵢ, dⱼ) | Sⱼ is reachable from Sᵢ } U
  ( (aᵢ, aᵢ) | (dᵢ, aᵢ) in S } U
  ( (aᵢ, s) | (dᵢ, aᵢ) in S } U
- l( aᵢ, dⱼ ) = 1 and all other edges 0
- c(e) = 1 for all edge e in E

Claim: There is a flight schedule of using k planes iff there is a flow f of G with v(f) = k.