CS:4350 Logic in Computer Science

Practice Questions (100 points)

1. (30 points) Given the following set of clauses,
   (p | q | s), (p | -r | -s), (-p | s), (q | r | s), (-p | -q | -r), (-q | r | -s)
   where "-" means negation, please answer the following questions:
   (a) Using the orders of p, q, r, s for splitting (the value 1 is used first for each splitting variable) in DPLL, what will be the truth values of variables, at what levels, when a model is found? What will be the head/tail positions of the clauses if the initial positions are the first and last positions?
   (b) What clauses can be learned if the CDCL technique is used during the execution of DPLL in (a)?
   (c) If we modify DPLL so that it will produce all the models of the clause set, how many models will be found and in what order if the splitting order is the same as in (a)?
   (d) If we feed the clause set to a SAT solver which works as a black box and can produce only one model at a time, what clauses should be added into the clause set so that the SAT solver will produce all models of the clause set?

2. (20 points) In number theory, **Fermat's Last Theorem** states that no three positive integers a, b, and c satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2.
   a. Please express Fermat's Last Theorem in the first-order logic using the following predicates: int, add, exp, where int(x) = true iff x is an integer, add(x, y, z) = true iff x+y=z, exp(x, y, z) = true iff $x^y = z$, and the usual relations over integers, such as >.
   b. Please convert the negation of the formula in (1) into clausal form.

3. (20 points) For the following formula of the first order logic, either prove formally that it is valid or give a falsifying interpretation:
   $$(\forall x\ p(x) \leftrightarrow \forall x\ q(x)) \rightarrow \forall x\ (p(x) \leftrightarrow q(x)).$$

4. (30 points) Please write a Prolog program myUnify(T1, T2, U), which takes T1 and T2, two terms built on constants a, b, binary function f, unary function g, and variables x, y, and z, and computes the most general unifier of T1 and T2 in U (as a list of pair(variable, term)) if T1 and T2 are unifiable.