1. (25) Prove that the class of Turing-recognizable languages are closed under union, concatenation, but not under complementation.

2. (15) In the proof of the Cook-Levin theorem, a window is a $2 \times 3$ rectangle of cells. Show why the proof would have failed if we had used $2 \times 2$ windows instead.

3. (35) Let $S = \{\langle M \rangle \mid M$ is a TM and $L(M) = \{\langle M \rangle \}\}$. Prove formally that $S$ is undecidable, though $S$ is Turing-recognizable.

4. (25) Let $\text{Bin-Packing} = \{\langle S, k \rangle \mid S = \{x_i \mid 1 \leq i \leq n, 0 < x_i < 1\}$ and $k$ is a positive integer, such that $S$ can be partitioned into $S = S_1 \cup S_2 \cup \ldots \cup S_k$ and $\sum_{y \in S_j} y \leq 1$ for $1 \leq j \leq k \}$. Show that (a) $\text{Bin-Packing}$ is in NP; (b) $\text{Subset-Sum} \leq_P \text{Bin-Packing}$. 