CS 4330 HW4 Solution

2.4

b)

Right linear G:

$S \longrightarrow$ 0A | 1B | ϵ

A $\longrightarrow$ 0A | 1A | 0

B $\longrightarrow$ 0B | 1B | 1

Left linear is obtained by reversing the rhs of G.

c)

Right linear G:
$S \longrightarrow 0|1|00S|01S|10S|11S$
Left linear is obtained by reversing the rhs of G.

e)

$$S \longrightarrow 0S0|1S1|0|1|\epsilon$$

2.6

b)

$$S \rightarrow XbXaX|T|U$$

$$T \rightarrow aTb|Tb|b$$

$$U \longrightarrow aUb|aU|a$$

$$X \longrightarrow a|b|aX|bX|\epsilon$$

d)

$$S \to UPV$$
$$P \to aPa|bPb|T|\epsilon$$
$$T \longrightarrow \#MT|\#$$
$$U \longrightarrow M\#U|\epsilon$$
$$V \longrightarrow MV|\epsilon$$
$$M \longrightarrow aM|bM|\epsilon$$

2.9

Let $G = (V, \Sigma, R, S)$ be a CFG that recognizes A where $V = \{S, E_{ab}, E_{bc}, C, A\}$ and Σ is $\{a, b, c\}$. The rules for G are

$$S \to E_{ab}C|AE_{bc}$$
$$E_{ab} \to aE_{ab}b|\epsilon$$
$$E_{bc} \longrightarrow bE_{bc}c|\epsilon$$
$$C \longrightarrow Cc|\epsilon$$
$$A \longrightarrow Aa|\epsilon$$

Consider derivations for ϵ using G. On one hand we can choose $E_{ab}C$ and then generate ϵ from both E_{ab} and C. On the other hand, we can choose AE_{bc} and get ϵ from both E_{bc} and A. Thus G is ambiguous.

2.14

First, add new start variable S_0 and new rule $S_0 \to S$,

$$S_0 \to S$$
$$S \to BSB|B|\epsilon$$
$$B \longrightarrow 00|\epsilon$$

Removing ϵ-rules

$$S_0 \to S|\epsilon$$
$$S \to BSB|B|BS|SB|S|BB$$
$$B \longrightarrow 00$$

Then remove unit rules $S \longrightarrow S$

$$S_0 \to S|\epsilon$$
$$S \to BSB|B|BS|SB|BB$$
$$B \longrightarrow 00$$

Removing unit rules $S \longrightarrow B$

$$S_0 \to S|\epsilon$$
$$S \to BSB|00|BS|SB|BB$$
$$B \longrightarrow 00$$

Removing unit rules $S_0 \longrightarrow S$

$$S_0 \to BSB|BS|SB|BB|\epsilon$$
$$S \to BSB|BS|SB|BB$$
$$B \longrightarrow 00$$

Replace 0 by variable C gives

$$S_0 \to BD|BS|SB|BB|\epsilon$$
$$S \to BD|BS|SB|BB$$
$$B \longrightarrow CC$$
$$C \longrightarrow 0$$
$$D \longrightarrow SB$$

2.16 Show that the class of context-free languages is closed under the regular operations, union, concatenation, and star.

Proof: Suppose L_1 and L_2 are generated by CFG $G_1 = (V_1, \Sigma, S_1, P_11)$ and $G_2 = (V_2, \Sigma, S_2, P_2)$, respectively. Then

$L_1 \cup L_2$ can be generated by CFG $G = (\{S\} \cup V_1 \cup V_2, \Sigma, S, \{ S \rightarrow S_1 | S_2 \} \cup P_1 \cup P_2)$

L_1L_2 can be generated by CFG $G = (\{S\} \cup V_1 \cup V_2, \Sigma, S, \{ S \rightarrow S_1S_2 \} \cup P_1 \cup P_2)$

L_1^* can be generated by CFG $G = (\{S\} \cup V_1 \cup V_2, \Sigma, S, \{ S \rightarrow S_1S \mid \varepsilon \} \cup P_1 \cup P_2)$

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