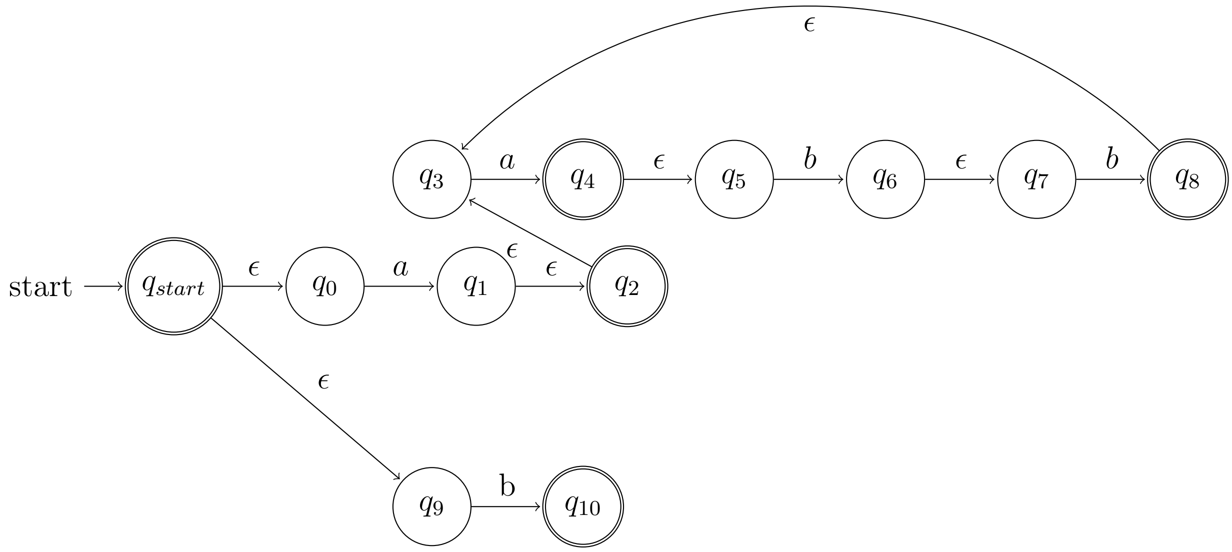
CS 4330 HW3 Solution

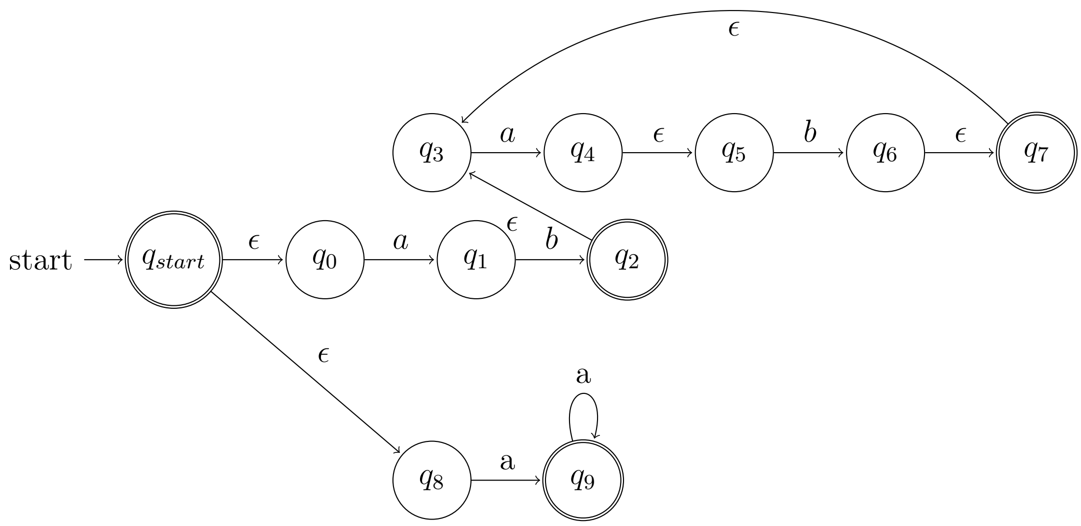
February 20, 2019

1.28

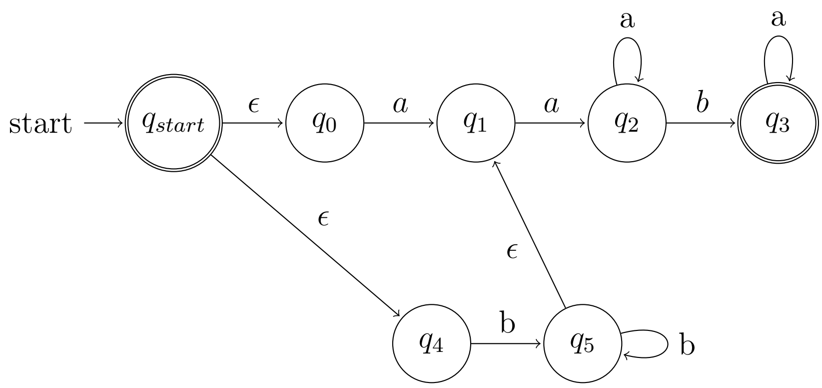
a)$a(abb)^* \cup b$



b) $a^+ \cup (ab)^+$



c) $(a \cup b^+)a^+b^+$

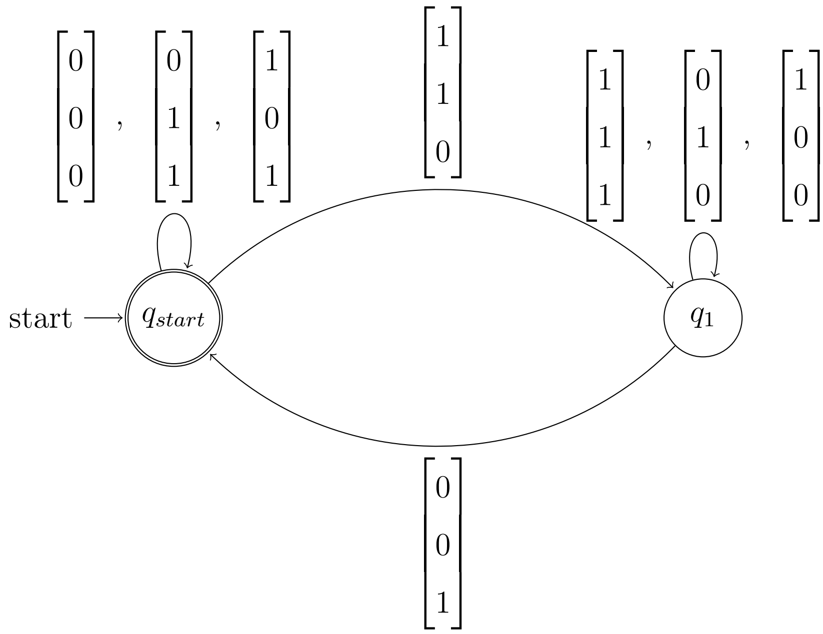


1.29(b)

Suppose for contradiction that A_2 is a regular language. Let p be the pumping length of A_2 and $w = a^p ba^p ba^p b$ where $|w| \geq p$. By pumping lemma, we can split w into 3 parts x, y, z. By (3) conditions of pumping lemma, $|xy| \leq p$ or x, y consists only a's. By (2) condition, $|y| > 0$ so y must contain at least 1 a. Let $y = a^k$ for some $k \geq 1$. Then by (1), we know that $xy^i z \in A_2$ for all $i \geq 0$. Suppose $i = 2$, then $xy^2 z = a^{p+k} ba^p ba^p b \notin A_2$, which is a contradiction. Therefore, A_2 is not a regular language.

1.32

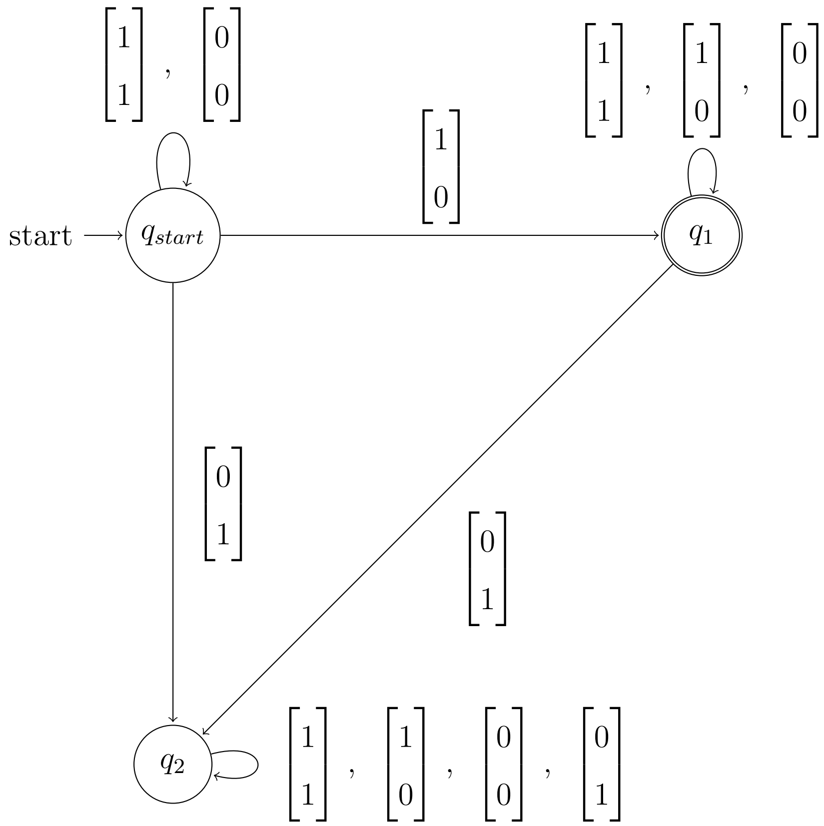
To prove B is regular, we will prove that $B^{\mathbb{R}}$ is regular. Let M be a DFA recognizing $B^{\mathbb{R}}$ as follows



Since $B^{\mathbb{R}}$ is regular and regular language closed under reverse operation, B is regular

1.34

To prove D is regular, let M be a DFA recognizing D as follows



1.41

$Shuffle(A,B) = \{w | w = a_1 b_1 ... a_k b_k,$ where $a_1...a_k \in A$ and $b_1...b_k \in B,$ each $a_i, b_i \in \Sigma\}$

Let $M_A = (Q_A, \Sigma, \delta_A, q_A, F_A)$ and $M_B = (Q_B, \Sigma, \delta_B, q_B, F_B)$ be DFA that recognize language A and B respectively. To prove that the class of regular languages closed under perfect shuffle, let's construct DFA $M_{Shuffle} = (Q_{Shuffle}, \Sigma, \delta_{Shuffle}, q_{Shuffle}, F_{Shuffle})$ that recognize $Shuffle(A,B)$

Since $M_{Shuffle}$ need to switch between state in M_A and M_B, $Q_{Shuffle} = Q_A \times Q_B \times \{odd, even\}$. The odd characters are for the character in A, and even characters are for the strings in B. the start state will be the start state of M_A, which is (q_A, q_B, odd).

The transitions will be defined as follows

$$\delta((q_a, q_b, odd), x) = (\delta_A(q_a, x), q_b, even)$$

$$\delta((q_a, q_b, even), x) = (q_a, \delta_B(q_b, x), odd)$$

The accept states of $M_{Shuffle}$ will be in even position and must be in accept states of M_A and M_B

$$F(M_{Shuffle}) = \{(q_a, q_b, even) | q_a \in F_A, q_b \in F_B\}$$

1.42

$Shuffle(A,B) = \{w | w = a_1 b_1 ... a_k b_k,$ where $a_1...a_k \in A$ and $b_1...b_k \in B,$ each $a_i, b_i \in \Sigma^*\}$

Let $M_A = (Q_A, \Sigma, \delta_A, q_A, F_A)$ and $M_B = (Q_B, \Sigma, \delta_B, q_B, F_B)$ be DFA that recognize language A and B respectively. The difference of this problem from 1.41 is that a_i and b_i can be of arbitrary length. We can adopt similar ideas to 1.41 by designing a machine that interleaves between states of M_A and M_B. However, each character can come from a string in A or B. Thus, we can create a NFA $M_{Shuffle}$ that nondeterministically go to M_A or M_B where

$Q_{Shuffle} = Q_A \times Q_B$

The new start state will be (q_A, q_b)

The accepting states would be $F_{Shuffle} = \{F_A \times F_B\}$ since accepted string is the interleaving of string from A and B.

The transition functions will be defined as

$\delta((q_a, q_b)x) = \{(\delta_A(q_a, x), \delta_B(q_b, x)\}$

Since $M_{Shuffle}$ recognizes $Shuffle(A, B)$, $Shuffle(A, B)$ is regular.

1.46

a) $A = \{0^n 1^m 0^n\}$

Suppose for contradiction that A is a regular language. Let p be the pumping length of B and $w = 0^p 1 0^p$ where $|w| \geq p$. By pumping lemma, we can split w into 3 parts x, y, z. By (3) conditions of pumping lemma, $|xy| \leq p$ or x, y consists only 0's. By (2) condition, $|y| > 0$ so y must contain at least a 0. Let $y = 0^k$ for some $k \geq 1$. Then by (1), we know that $xy^i z \in A$ for all $i \geq 0$. Suppose $i = 2$, then $xy^2z = 0^{p+k}10^p \notin A$, which is a contradiction. Therefore, A is not regular.

c) $A = \{w | w \in \{0, 1\}^* \text{ is not a palindrome}\}$

Since regular language is closed under complement, if we can prove that complement of A is not regular, it follows that A is not regular. Suppose for contradiction that complement of A is a regular language. Let p be the pumping length and $w = 0^p 1 0^p$ where $|w| \geq p$. From part a, we proved that w violates pumping lemma. Thus, A is not regular.

1.47

The language is not even context-fre; see Problem 2.42.