CS 4330

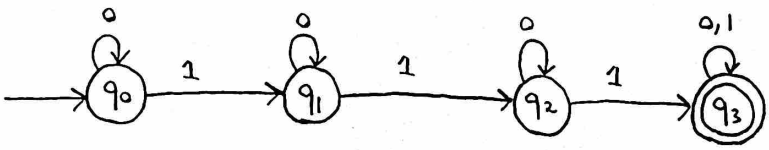
Theory of Computation

Homework 2 Solutions

1.10 a)

1.6 b) $L = \{ w \mid w \text{ contains at least three 1's} \}$

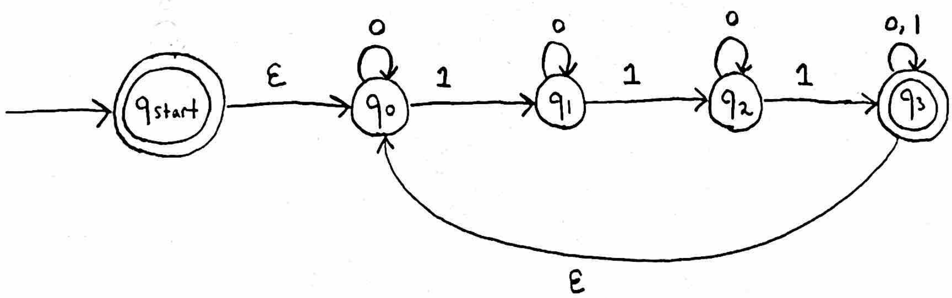
⇒ Let N be the NFA that recognizes L



Want to build an NFA that recognizes L^* using Theorem 1.49

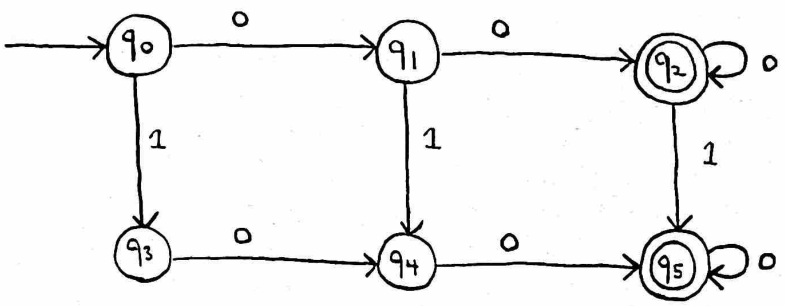
⇒ Add a new start state that is also a final state

⇒ Draw epsilon transitions from all final states to old start state



1.10 b) 1.6 j) $L = \{ w \mid w \text{ contains at least two 0's and at most one 1} \}$

⇒ Let N be the NFA that recognizes L



Want to build an NFA that recognizes L^* using Theorem 1.49

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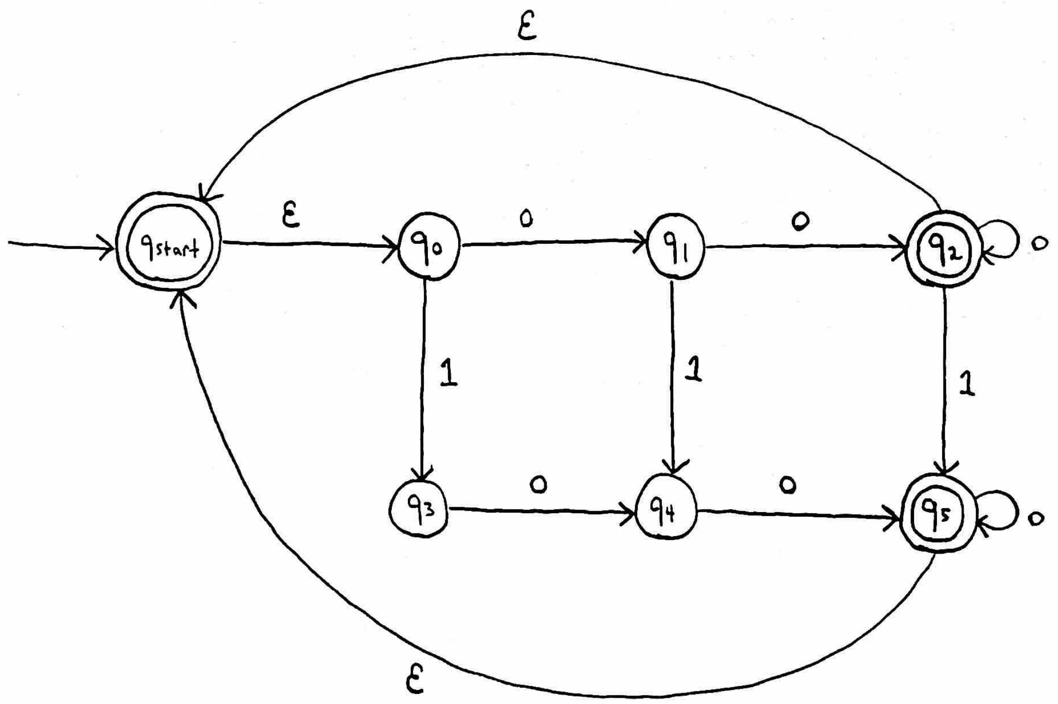
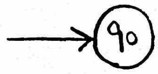


1.10 c)

1.6 m) $L =$ The empty set

$\Rightarrow L = \emptyset$

$\Rightarrow$ Let N be the NFA that recognizes L

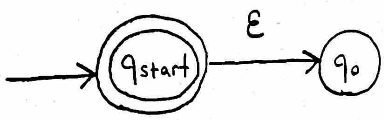


(Please see Lemma 1.55 on page 67, #3)

Want to build an NFA that recognizes L^* using Theorem 1.49

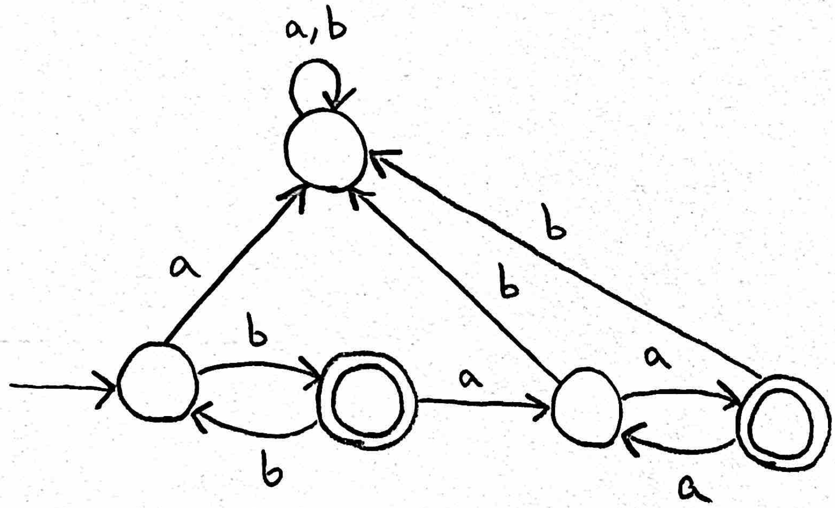
$\Rightarrow$ Add a new start state that is also a final state

$\Rightarrow$ Draw epsilon transitions from all final states to old start state



1.12

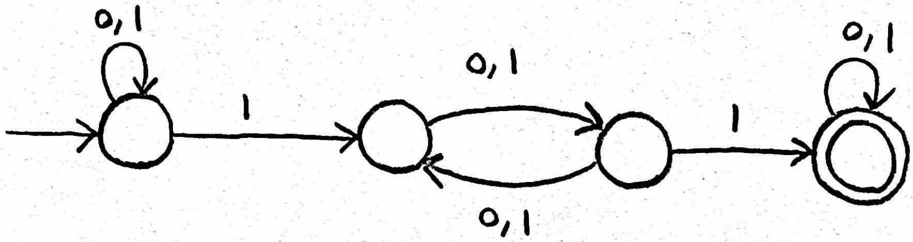
④



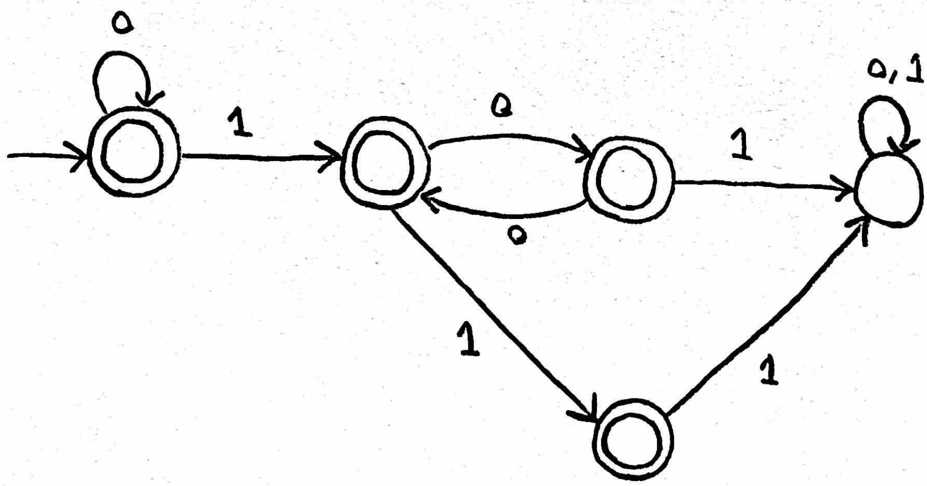
RE = $b(bb)^*(aa)^*$

1.13

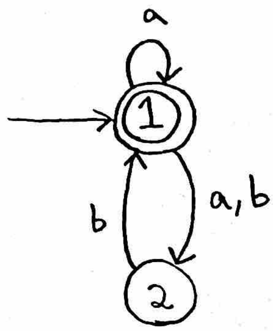
4-State NFA for $\overline{F}$



DFA for F



1.16 a)



Want to convert the following NFA to an equivalent DFA Using Theorem 1.39

NFA $N = (Q, \Sigma, \delta, q_0, F)$

1. Set of states for N: $Q = \{1, 2\}$

2. Input alphabet: $\Sigma = \{a, b\}$

3. Transition Function δ:

δ	a	b
1	$\{1,2\}$	$\{2\}$
2	$\emptyset$	$\{1\}$

$\Rightarrow \delta(1, a) = \{1, 2\}$, $\delta(1, b) = \{2\}$

$\Rightarrow \delta(2, a) = \{\}$, $\delta(2, b) = \{1\}$

4. Start state for N: $q_0 = 1$

5. Set of final states for N: $F = \{1\}$

Using NFA $N = (Q, \Sigma, \delta, q_0, F)$ want to construct an equivalent DFA $M = (Q', \Sigma, \delta', q_0', F')$ using Theorem 1.39 on page 55

1.16 a) Continued

For DFA $M = (Q', \Sigma, \delta', q_0', F')$

1. Set of states for M:

$$Q' = P(Q)$$
$$= P(\{1,2\})$$
$$= \{\{1,2\},\{1\},\{2\},\{\}\}$$

2. Transition Function δ':

$$\delta'(R,a) = \bigcup_{r \in R} \delta(r,a)$$

$$\Rightarrow \delta'(\{1,2\},a) = \delta(1,a) \cup \delta(2,a)$$
$$= \{1,2\} \cup \{\}$$
$$= \{1,2\}$$

$$\Rightarrow \delta'(\{1,2\},b) = \delta(1,b) \cup \delta(2,b)$$
$$= \{2\} \cup \{1\}$$
$$= \{1,2\}$$

$$\Rightarrow \delta'(\{1\},a) = \delta(1,a)$$
$$= \{1,2\}$$

$$\Rightarrow \delta'(\{1\},b) = \delta(1,b)$$
$$= \{2\}$$

$$\Rightarrow \delta'(\{2\},a) = \delta(2,a)$$
$$= \{\}$$

$$\Rightarrow \delta'(\{2\},b) = \delta(2,b)$$
$$= \{1\}$$

1.16 a) Continued

Transition table for DFA

δ'	a	b
$\{1\}$	$\{1,2\}$	$\{2\}$
$\{2\}$	$\{\ \}$	$\{1\}$
$\{1,2\}$	$\{1,2\}$	$\{1,2\}$
$\{\ \}$	$\{\ \}$	$\{\ \}$

(Trap State)

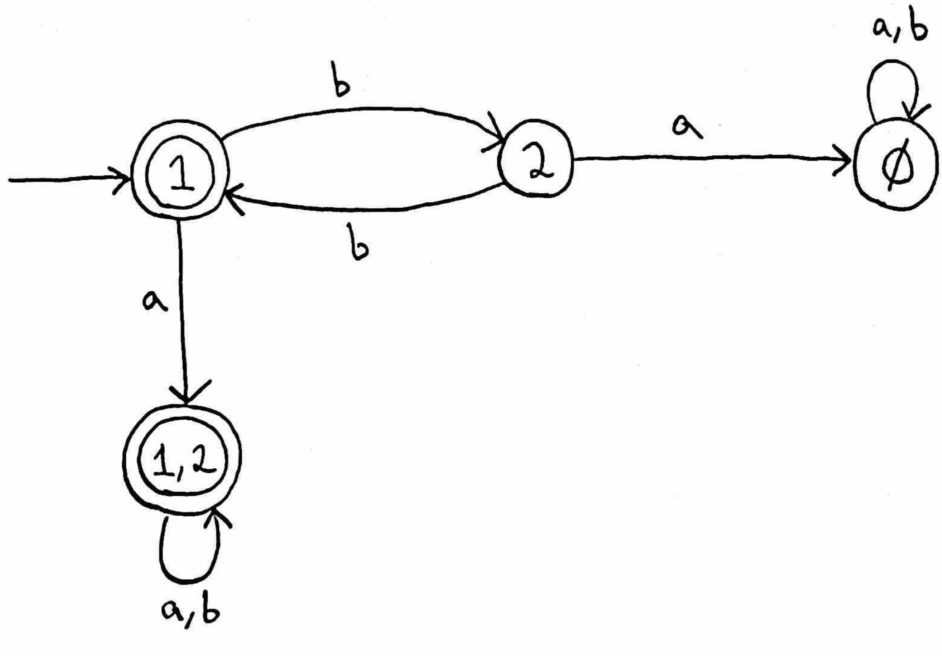
3. Start state for M:

$$q_0' = \{q_0\}$$
$$= \{1\}$$

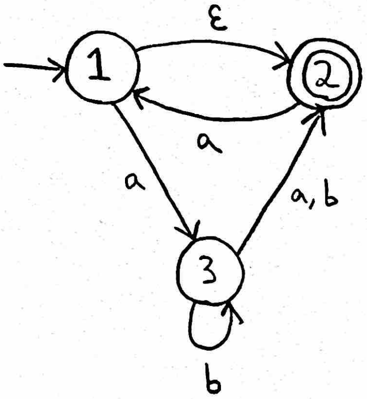
4. Set of final states for M:

$$F' = \{\{1\}, \{1,2\}\}$$

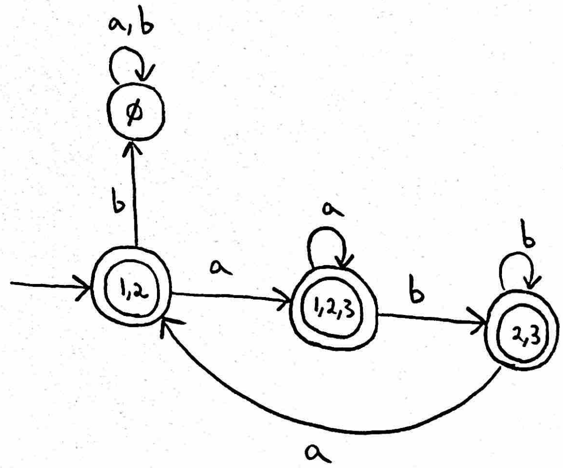
$\Rightarrow$ State diagram for DFA M:



1.16 b)



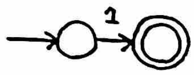
Theorem 1.39
NFA to DFA
$\Longrightarrow$



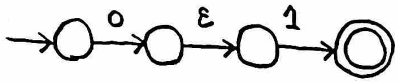
1.17 a) $R = (01 \cup 001 \cup 010)^*$

NFA for $R = 0$ (Lemma 1.55 #1)

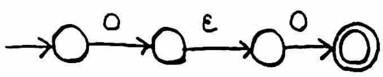
NFA for $R = 1$ (Lemma 1.55 #1)



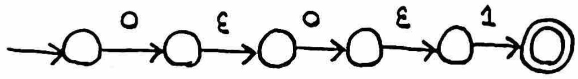
NFA for $R = 01$ (Theorem 1.47, Figure 1.48)



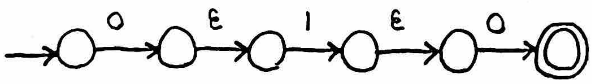
NFA for $R = 00$ (Theorem 1.47, Figure 1.48)



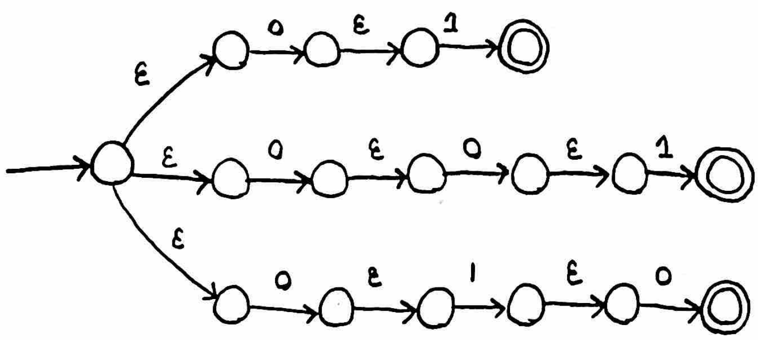
NFA for $R = 001$ (Theorem 1.47, Figure 1.48)



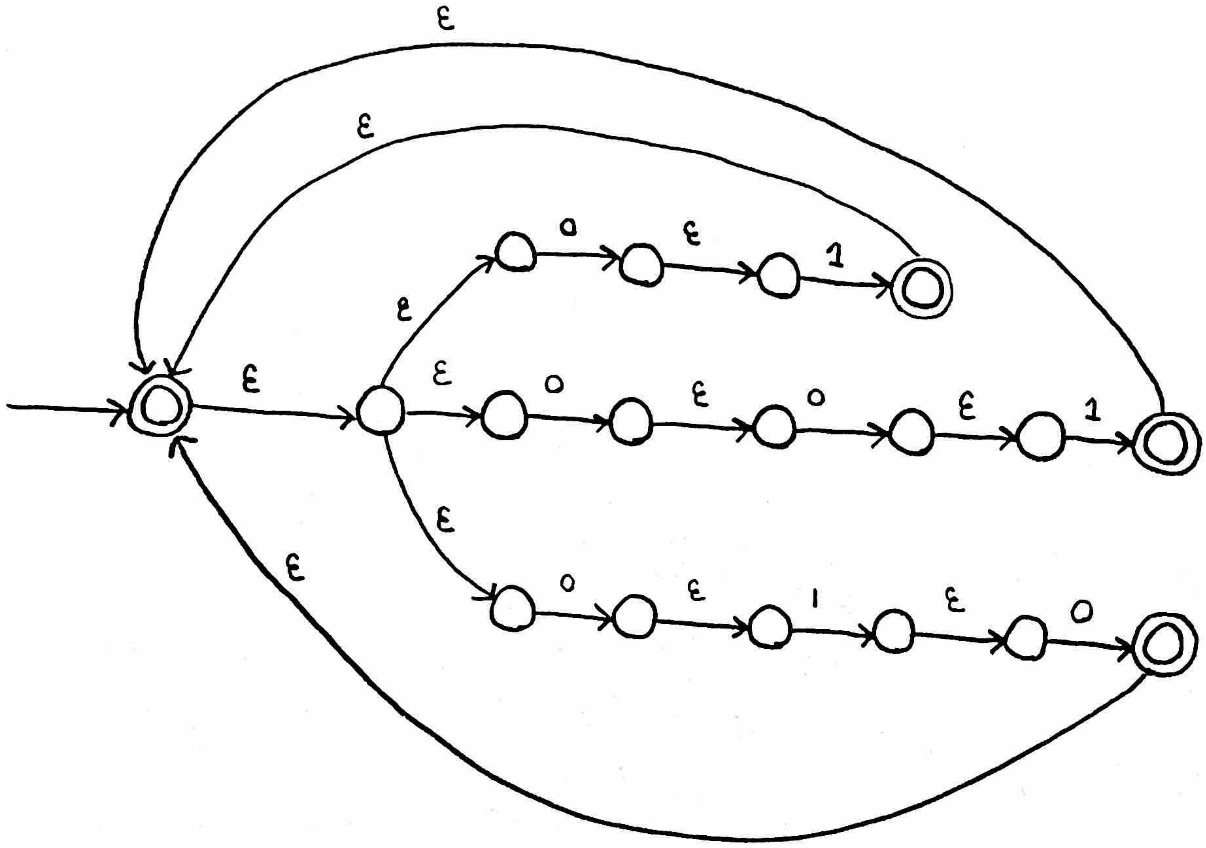
NFA for $R = 010$ (Theorem 1.47, Figure 1.48)



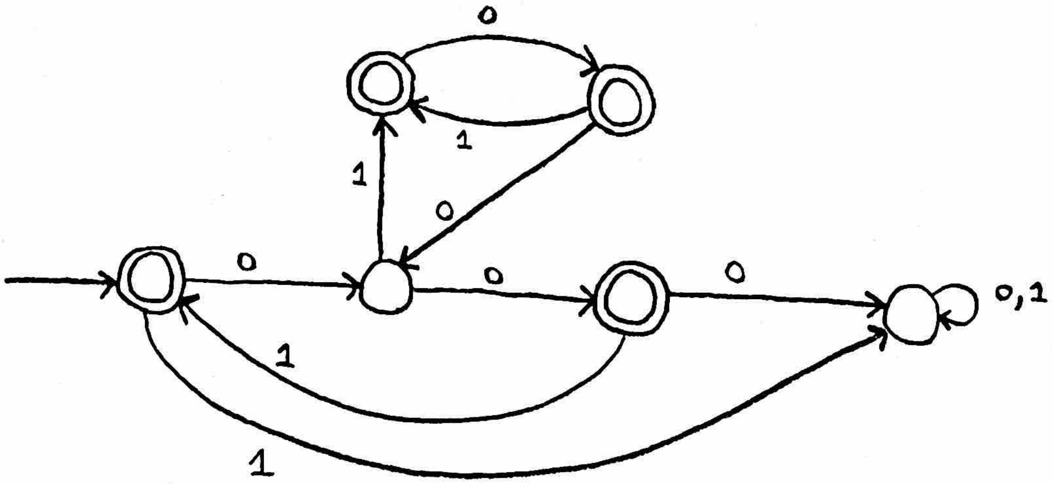
NFA for $R = (01 \cup 001 \cup 010)$ (Theorem 1.45, Figure 1.46)



NFA for $R = (01 \cup 001 \cup 010)^*$ (Theorem 1.49, Figure 1.50)

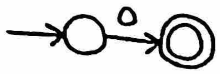


1.17 b)



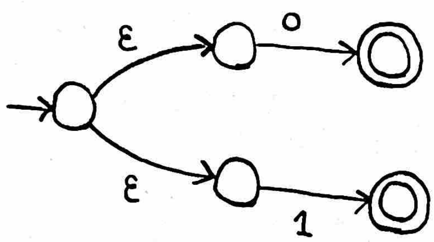
1.19 a) $R = (0 \cup 1)^* 000 (0 \cup 1)^*$

NFA for $R = 0$ (Lemma 1.55, #1)

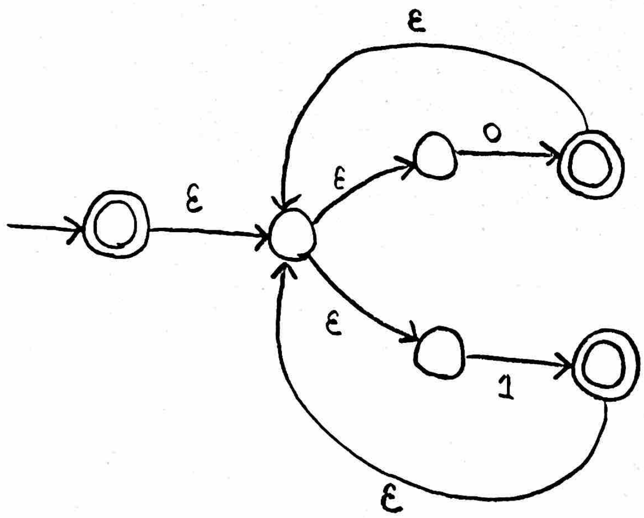


NFA for $R = 1$ (Lemma 1.55, #1)

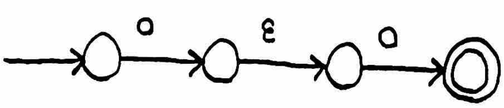
NFA for $R = (0 \cup 1)$ (Theorem 1.45, Figure 1.46)



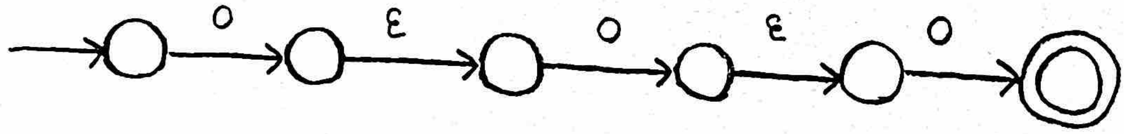
NFA for $R = (0 \cup 1)^*$ (Theorem 1.49, Figure 1.50)



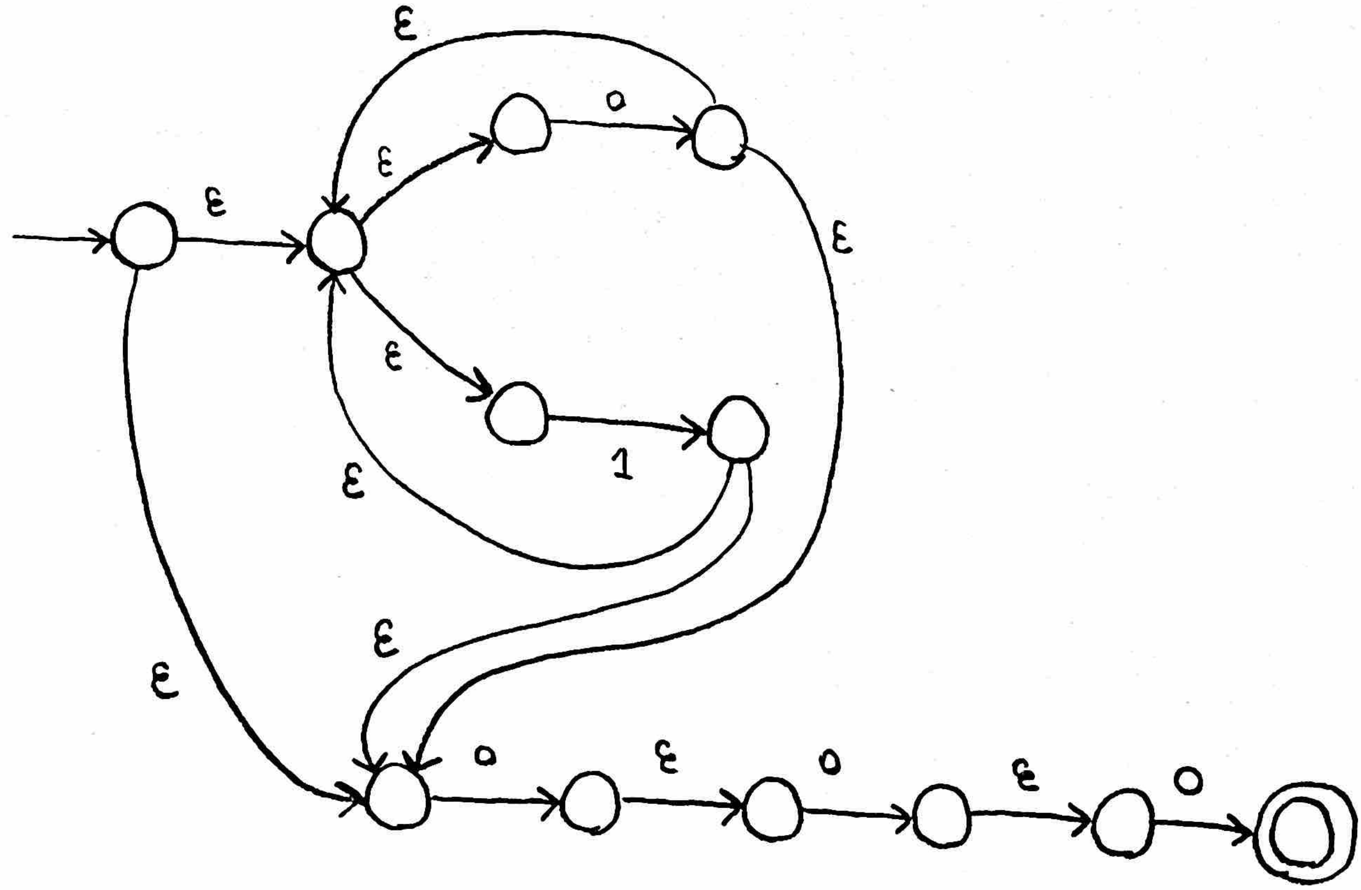
NFA for $R = 00$ (Theorem 1.47, Figure 1.48)



NFA for $R = 000$ (Theorem 1.47, Figure 1.48)

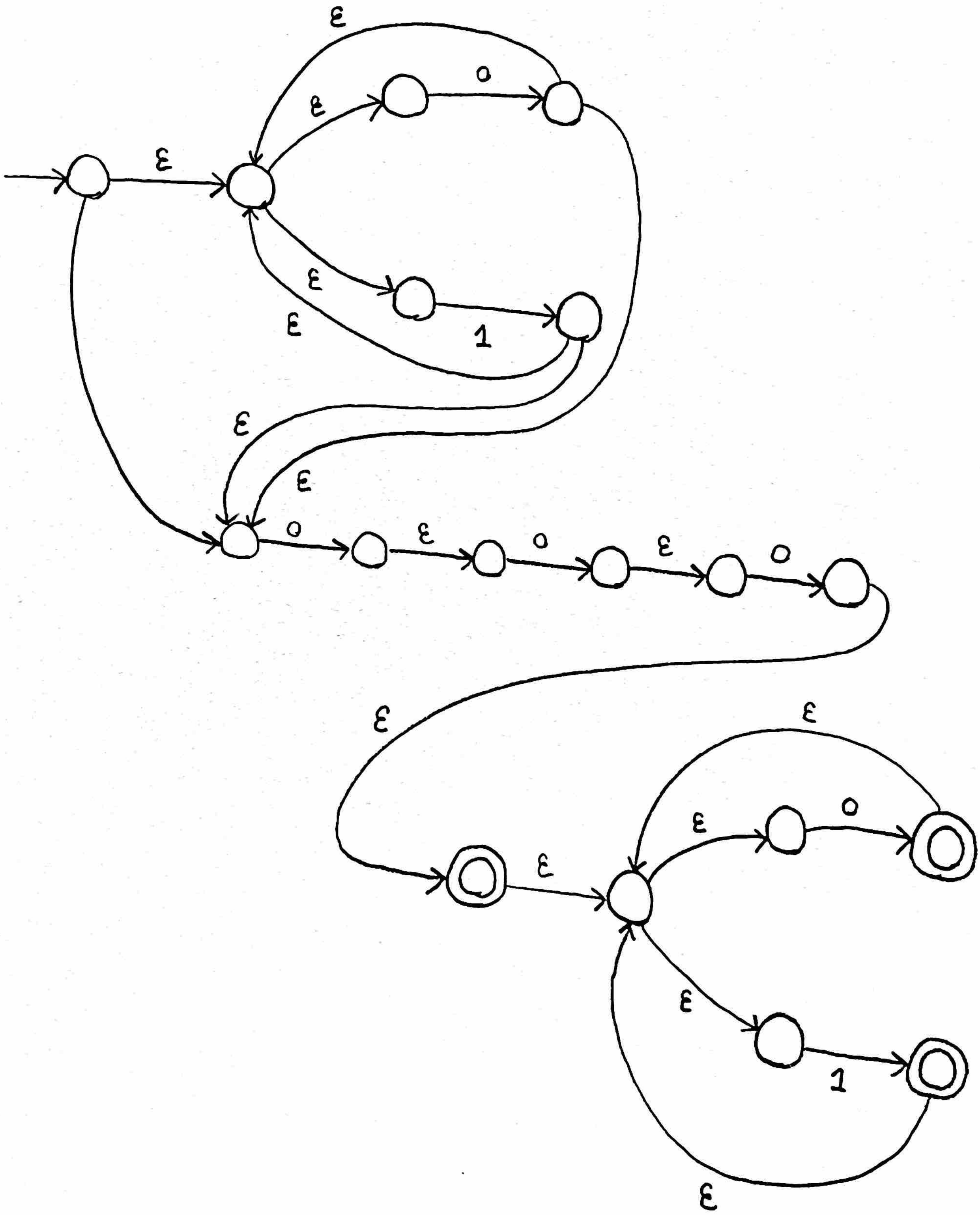


NFA for $R = (0 \cup 1)^* 000$ (Theorem 1.47, Figure 1.48)



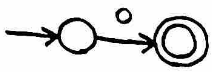
1.19 a) Continued

NFA for $R = (0 \cup 1)^* 000 (0 \cup 1)^*$ (Theorem 1.47, Figure 1.48)

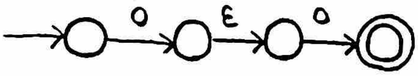


1.19 b) $R = (((00)^*(11)) \cup 01)^*$

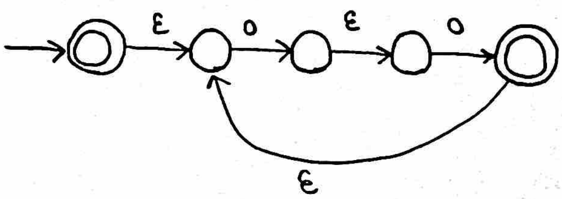
NFA for $R = 0$ (Lemma 1.55 # 1)



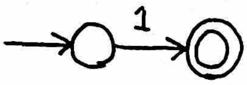
NFA for $R = 00$ (Theorem 1.47, Figure 1.48)



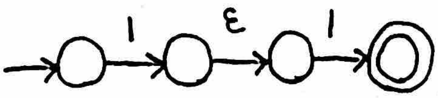
NFA for $R = (00)^*$ (Theorem 1.49, Figure 1.50)



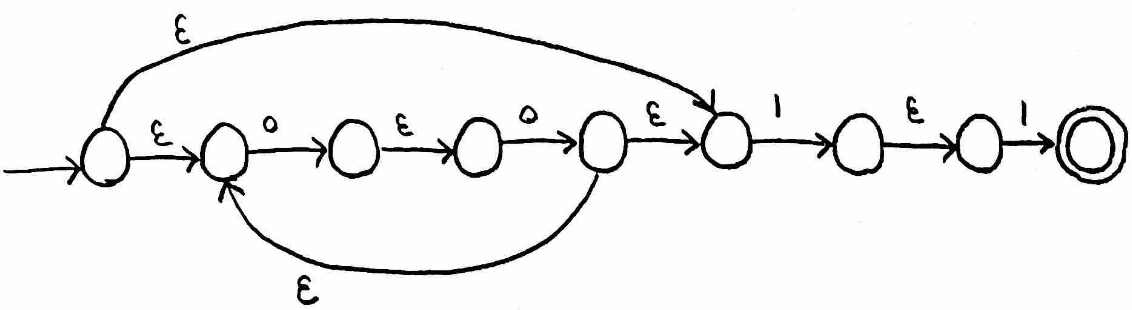
NFA for $R = 1$ (Lemma 1.55 # 1)



NFA for $R = 11$ (Theorem 1.47, Figure 1.48)



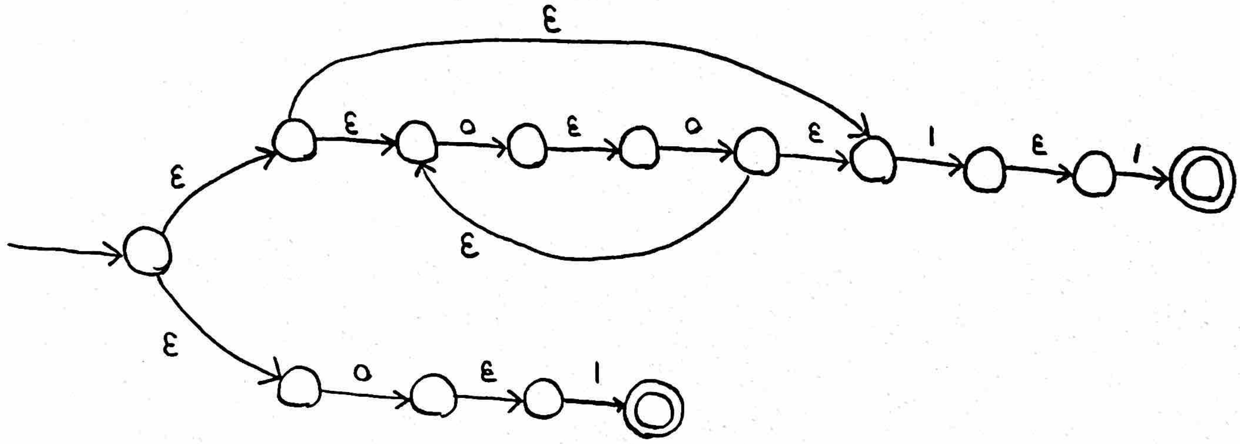
NFA for $R = (00)^*(11)$ (Theorem 1.47, Figure 1.48)



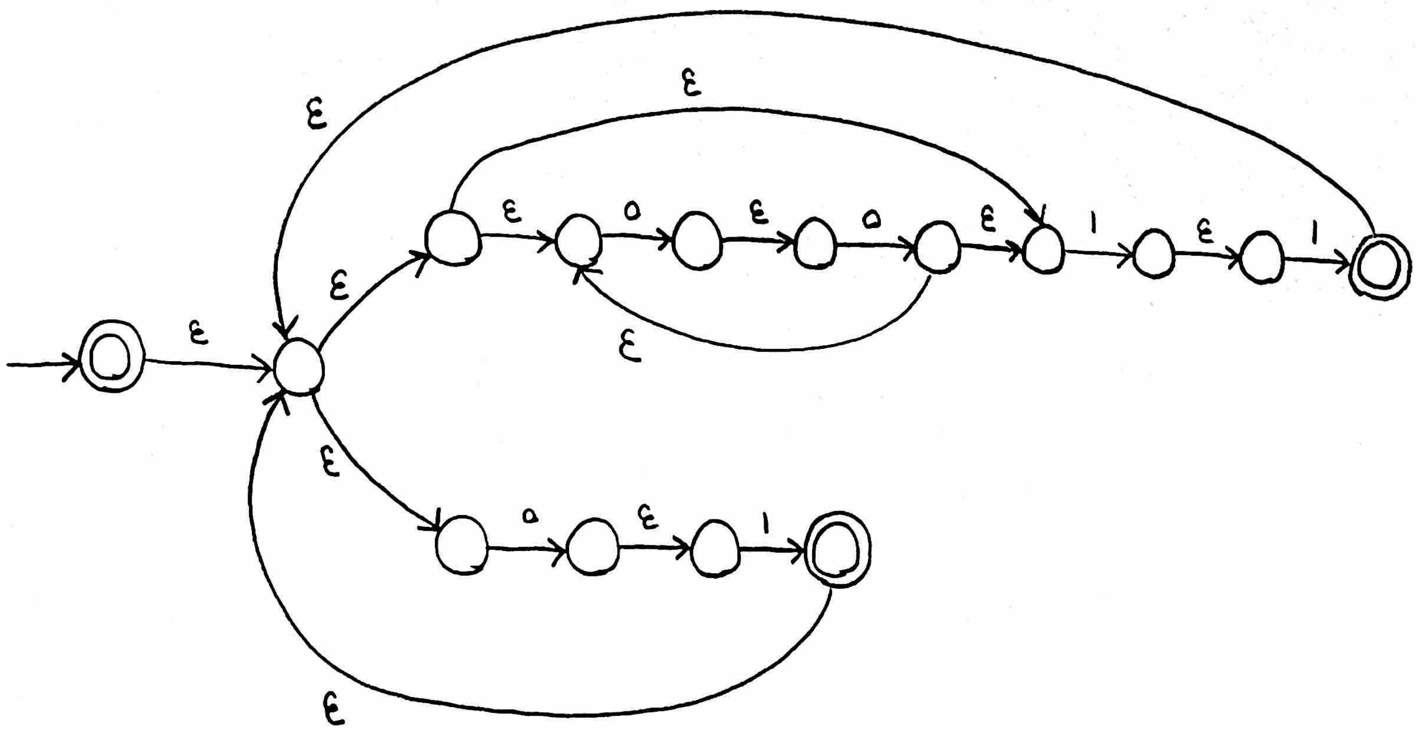
1.19b) Continued

NFA for R = 01 (Theorem 1.47, Figure 1.48)

NFA for R = ((00)* (11) ∪ 01) (Theorem 1.45, Figure 1.46)

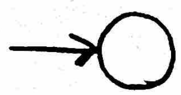


NFA for R = ((00)* (11) ∪ 01)* (Theorem 1.49, Figure 1.50)



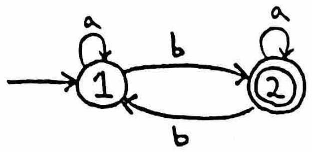
1.19 c) $R = \emptyset^*$

NFA for $R = \emptyset$ (Lemma 1.55, #3)



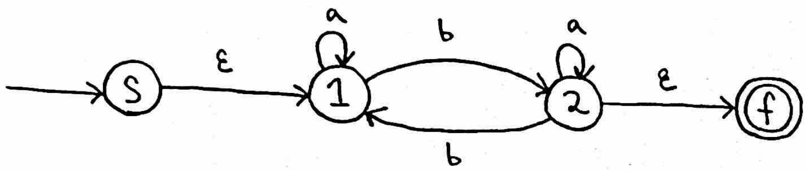
NFA for $R = \emptyset^*$ (Theorem 1.49, Figure 1.50)

1.21 a)



Want to convert the following DFA to an RE (Apply Lemma 1.60)

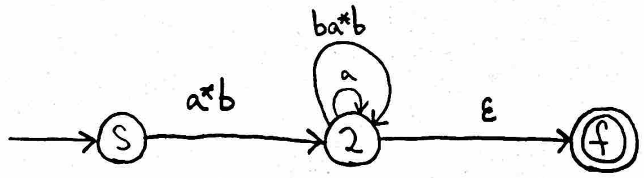
First convert the following DFA into an equivalent GNFA:



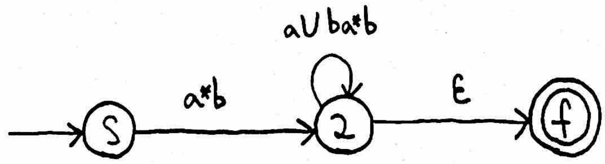
Eliminating state 1:

1) For the path $s \to 1 \to 2$: Create an edge from s to 2 labelled $\varepsilon a^*b = a^*b$

2) For the path $1 \to 2 \to 1$: Create an edge from 2 to 2 labelled ba^*b

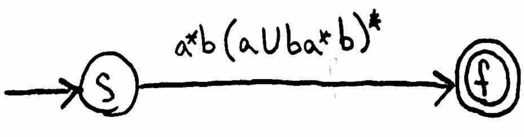


1) For the path $2 \to 2$: Combine both edges by taking the union $\Rightarrow a \cup ba^*b$



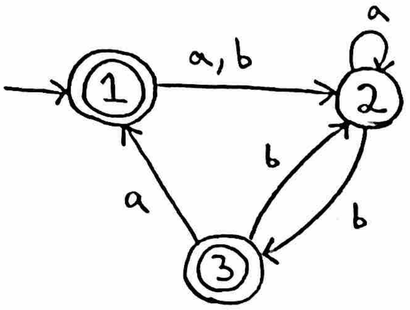
Eliminating state 2:

1) For the path $s \to 2 \to f$: Create an edge labelled $a^*b(a \cup ba^*b)^*\varepsilon = a^*b(a \cup ba^*b)^*$



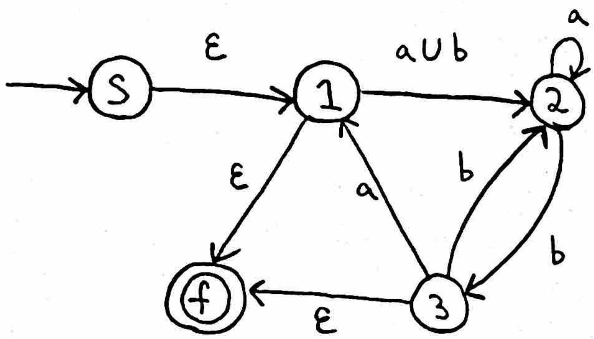
$\Rightarrow \boxed{a^*b(a \cup ba^*b)^*}$

1.21 b)



Want to convert the following DFA to a RE (Apply Lemma 1.60)

First convert the following DFA into an equivalent GNFA:



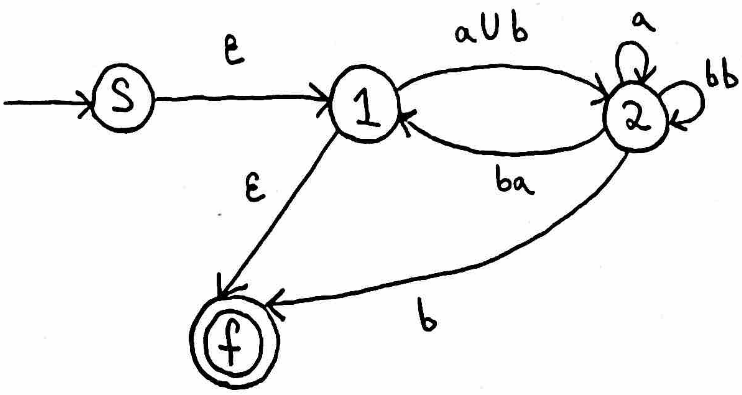
Next, eliminate states of GNFA one at a time until we are left with the start state and final state.

Eliminating state 3:

1) For the path $2 \rightarrow 3 \rightarrow 1$: Create an edge from 2 to 1 labelled ba

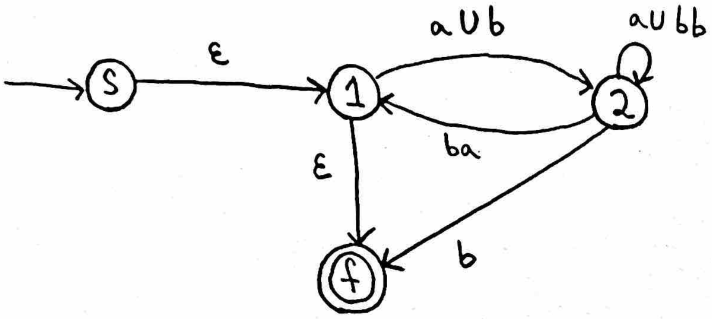
2) For the path $2 \rightarrow 3 \rightarrow 2$: Create an edge from 2 to 2 labelled bb

3) For the path $2 \rightarrow 3 \rightarrow f$: Create an edge from 2 to f labelled $b\varepsilon = b$



1.21 b) Continued

1) For the path 2→2: Combine both edges by taking the union ⇒ $a \cup bb$



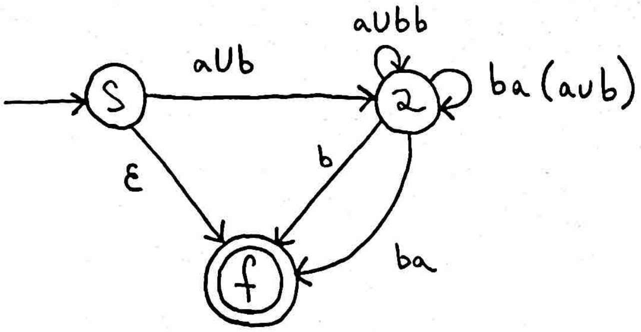
Eliminating state 1:

1) For the path $s \to 1 \to f$: Create an edge from s to f labelled $\varepsilon\varepsilon = \varepsilon$

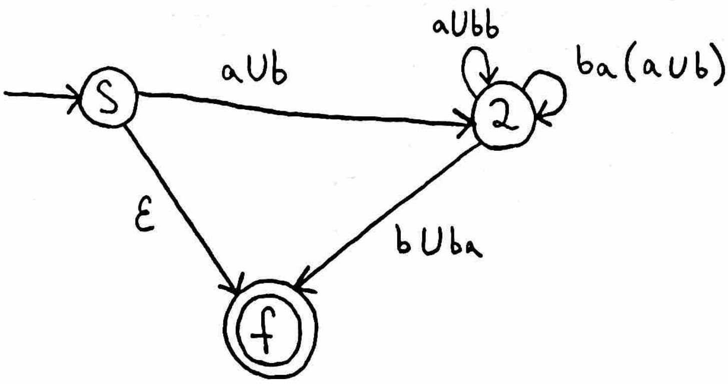
2) For the path $s \to 1 \to 2$: Create an edge from s to 2 labelled $\varepsilon(a \cup b) = a \cup b$

3) For the path $2 \to 1 \to f$: Create an edge from 2 to f labelled $ba\varepsilon = ba$

4) For the path $2 \to 1 \to 2$: Create an edge from 2 to 2 labelled $ba(a \cup b)$

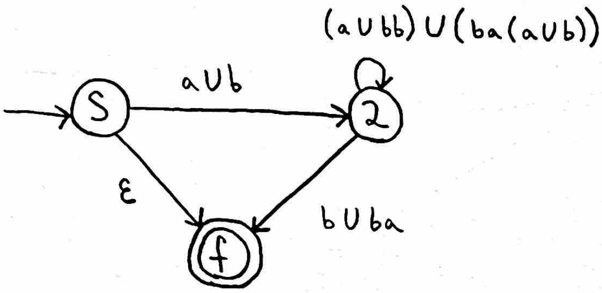


Combining both edges from 2 to f with union ⇒ $b \cup ba$



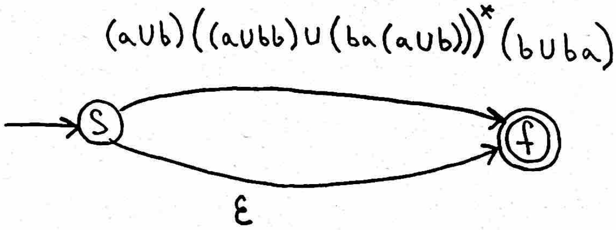
1.12b) Continued

Combining both edges from 2 to 2 with union $\Rightarrow (a \cup bb) \cup (ba(a \cup b))$

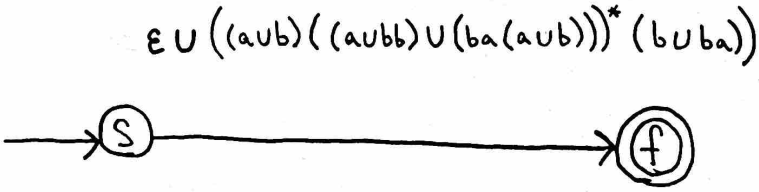


Eliminating state 2:

1) For path $s \rightarrow 2 \rightarrow f$, draw an edge from s to f labelled $(a \cup b)((a \cup bb) \cup (ba(a \cup b)))^*(b \cup ba)$



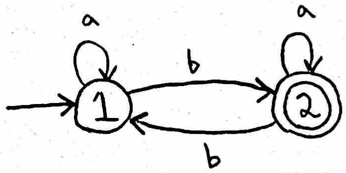
Combining both edges from s to f using union $\Rightarrow \varepsilon \cup ((a \cup b)((a \cup bb) \cup (ba(a \cup b)))^*(b \cup ba))$



$\Rightarrow$ $\boxed{\varepsilon \cup ((a \cup b)((a \cup bb) \cup (ba(a \cup b)))^*(b \cup ba))}$

1.21 Alternative Solutions (Produces different answers)

1.21 a)



Using substitution of "equal by equal" and the solution of $X = sX + t$ is $X = s^* t$

Define: $X_1 = aX_1 + bX_2$

$X_2 = aX_2 + bX_1 + \varepsilon$

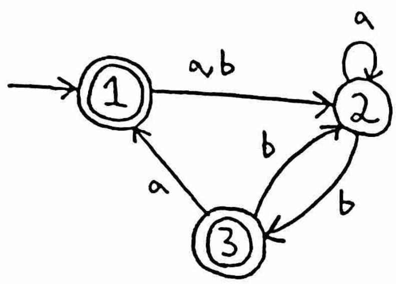
$\Rightarrow X_2 = a^*(bX_1 + \varepsilon)$

$= a^* bX_1 + a^*$

$\Rightarrow X_1 = aX_1 + ba^* bX_1 + ba^*$

$\Rightarrow X_1 = (a + ba^* b)^* ba^*$

1.21 b)



Using substitution of "equal by equal" and the solution of $X = sX + t$ is $X = s^*t$

Define: $X_1 = (a+b)X_2 + \varepsilon$

$X_2 = aX_2 + bX_3$

$X_3 = aX_1 + bX_2 + \varepsilon$

$\Rightarrow X_2 = aX_2 + b(aX_1 + bX_2 + \varepsilon)$

$= (a+bb)X_2 + baX_1 + b$

$\Rightarrow X_2 = (a+bb)^*(baX_1 + b)$

$= (a+bb)^* baX_1 + (a+bb)^* b$

$\Rightarrow X_1 = (a+b)((a+bb)^* baX_1 + (a+bb)^* b) + \varepsilon$

$= (a+b)(a+bb)^* baX_1 + (a+b)(a+bb)^* b + \varepsilon$

$\Rightarrow X_1 = ((a+b)(a+bb)^* ba)^* ((a+b)(a+bb)^* b + \varepsilon)$