CS:5810
Formal Methods in Software Engineering

Modeling in Alloy: Academia Model

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“Academia” Modeling Example

• We will model an academic enterprise expressing relationships between
  – People
    • Faculty
    • Students
      – Graduate
      – Undergraduate
    • Instructors – which can be grad students or faculty
  – Courses
  – Academic departments
  – Personal ID numbers

How should we model these basic domains in Alloy?
Strategy

• Build and validate your model incrementally
  – Start with basic signatures and fields
  – Add basic constraints
  – Instantiate the model and study the results
  – Probe the model with assertions
Strategy

• Add groups of features at a time
  – New signatures and fields
  – New constraints
  – Confirm previous assertions
  – Probe new features with assertions
Basic Components

• People
  – Students: Undergrads and Grads
  – Instructors: Faculty and Grads

• Courses

• Relationships
  – One instructor teaches a course
  – One or more students are taking a course
  – Students can be waiting for for course
abstract sig Person {}
sig Faculty extends Person {}
abstract sig Student extends Person {}
sig Graduate, Undergrad extends Student {}
sig Instructor in Person {}

sig Course {}
...

We are not specifying here that instructors can only be graduate students or faculty. We will do that later with a “fact” constraint.
Academia Fields

• One instructor teaches a course
• 2 choices:

```plaintext
sig Instructor in Person {
  teaches: Course
}

fact oneInstrucPerCourse {
  all c:Course | one teaches.c
}

sig Course {
  taughtby: one Instructor
}
```

We cannot specify that there is exactly one instructor per course

We have to add a fact specifying this constraint
Course Fields

- One *instructor teaches a course*
- One or more *students are taking a course*
- *Students* can be *waiting for for course*
Course Fields

- One instructor teaches a course
- One or more students are taking a course
- Students can be waiting for a course

```
sig Course {  
  taughtby: one Instructor,  
  enrolled: some Student,  
  waitlist: set Student  
}
```

- Exactly one instructor per course
- One or more students per course
- Zero or more students per course
More relations

• We may choose to define auxiliary relations:

  \textbf{teaches} (transpose of \textit{taughtby})
  \textbf{taking} (transpose of \textit{enrolled})
  \textbf{waitingfor} (transpose of \textit{waitlist})

\textbf{fun} \textbf{teaches}: Instructor \to Course \{ \sim \textit{taughtby} \}
\textbf{fun} \textbf{taking}: Student \to Course \{ \sim \textit{enrolled} \}
\textbf{fun} \textbf{waitingfor}: Student \to Course \{ \sim \textit{waitlist} \}

• Or not:

  \text{if } i \text{ is an instructor, then}
  \quad i.\textbf{teaches} \iff \textit{taughtby}.i
Note

• Let \( i \) be an Instructor

• Let \( taughtby \) be the following binary relation
  – \( taughtby: \) Course \( \rightarrow \) one Instructor

• The following expressions are equivalent and give a set of courses as result
  – \( taughtby.i \)
  – \( i.\sim taughtby \)
  – \( i[taughtby] \)
Academia Constraints

• All instructors are either faculty or graduate students
  – Was not expressed in set definition--although it could have, with
    \texttt{sig Instructor in Graduate + Faculty}

• No one is waiting for a course unless someone is enrolled

• No graduate students teach a course that they are enrolled in
Academia Constraints

fact  { 
    -- All instructors are either Faculty or Graduate Students

    -- no one is waiting for a course unless someone is enrolled
    -- (This is actually superfluous. Why?)

    -- graduate students do not teach courses they are enrolled in
    or waiting to enroll in
}

Academia Constraints

fact {
  -- All instructors are either Faculty or Graduate Students
  all i: Instructor | i in Faculty + Graduate

  -- no one is waiting for a course unless someone is enrolled
  -- (This is actually superfluous. Why?)
  all c: Course |
    some c.waitlist => some c.enrolled

  -- graduate students do not teach courses they are enrolled in
  or waiting to enroll in
  all c: Course |
    c.taughtby !in c.enrolled + c.waitlist
}

Academia *Realism* Constraints

- There is a *graduate* student who is an *instructor*

- There are at least:
  - Two *courses* and
  - Three *undergraduates*
Academia *Realism* Constraints

Can be added to the model as facts, or just put in a `run` command to instruct the Alloy Analyzer to ignore unrealistic instances

```alloy
pred RealismConstraints [] {
  -- there is a graduate student who is an instructor
  some Graduate & Instructor

  -- there are at least two courses
  #Course > 1

  -- there are at least three undergraduates
  #Undergrad > 2
}
```
Academia Assertions

Let’s check if our model has these properties:

• No *instructor* is on the waitlist for a *course* that he/she teaches

• No *student* is enrolled and on the waitlist for the same *course*
Academia Assertions

-- no instructor is on the waitlist for a course that he/she teaches

-- no student is enrolled and on the waitlist for the same course
-- no instructor is on the waitlist for a course that he/she teaches

```prolog
assert NoWaitingTeacher { 
    all c: Course |
    no (c.taughtby & c.waitlist)
}
```

-- no student is enrolled and on the waitlist for the same course

```prolog
assert NoEnrolledAndWaiting { 
    all c: Course |
    no (c.enrolled & c.waitlist)
}
```
Exercises

• Load academia-1.als

• With realism conditions enabled, do any instances exist in the default scopes?
  – Manipulate the scopes as necessary to obtain an instance under the realism conditions

• By looking at various sample instances, do you consider the model to be underconstrained in any way?

• Check assertions
Realism constraints

• No instances exist in the default scope

• Why?

  – default scope:
    at most 3 tuples in each top-level signature

  – entails: at most 3 Students

  – some Graduate & Instructor
    #Undergrad > 2

  – entails: at least 4 Students
**Realism Constraints**

```plaintext
pred [] RealismConstraints {
    -- there is a graduate student who's an instructor
    some Graduate & Instructor

    -- there are at least two courses
    #Course > 1

    -- there are at least three undergraduates
    #Undergrad > 2
}

run RealismConstraints for 4
```
Instance

#Undergrad > 2 #Undergrad > 1

Instance found:

Signatures:
- Course = \(\{C0,C1\}\)
- Person = \(\{U0,U1,G\}\)
- Faculty = \(\{\}\)
- Student = \(\{U0,U1,G\}\)
- Undergrad = \(\{U0,U1\}\)
- Graduate = \(\{G\}\)
- Instructor = \(\{G\}\)

Relations:
- taughtby = \(\{(C0,G),(C1,G)\}\)
- enrolled = \(\{(C0,U1),(C1,U0)\}\)
- waitlist = \(\{(C1,U1),(C1,U0)\}\)

Need to relate enrollment and waiting lists
Counter-example to assertion

Analyzing NoEnrolledAndWaiting ...

Counterexample found:

Signatures:
  Course = \{C\}
  Person = \{G0,G1,F\}
  Faculty = \{F\}
  Student = \{G0,G1\}
  Undergrad = \{}
  Graduate = \{G0,G1\}
  Instructor = \{G0,G1\}

Relations:
  taughtby = \{(C,G0)\}
  enrolled = \{(C,G1)\}
  waitlist = \{(C,G1)\}
Acadia Assertions

• No *student* is enrolled and on the waitlist for the same *course*
  
  – A counterexample has been found, hence we transform this assertion into a fact

• No *instructor* is on the waitlist for a *course* that he/she teaches
  
  – No counterexample
Academia Assertions

- **NoWaitingTeacher** assertion
  - No counterexample within the default scope
  - No counterexample within the scope 4, 5, 6, 10

- Can we conclude that the assertion is valid?
  - No! (It might have counterexamples but out of scope)

- But we take comfort in the
  - **small scope hypothesis**: if an assertion is not valid, it probably has a small counter-example
Why $\text{NoWaitingTeacher}$ holds

• Assertion

-- no instructor is on the waitlist for a course that he/she teaches

\[
\text{assert } \text{NoWaitingTeacher} \{ \\
\quad \text{all } c: \text{Course } | \text{ no } (c.\text{taughtby} \land c.\text{waitlist}) \\
\} \\
\]

• Facts

-- (i) faculty are not students and (ii) graduate students do not

-- teach courses they are enrolled in or waiting to enroll in

\[
\text{all } c: \text{Course } | \\
\quad c.\text{taughtby} \not\in c.\text{enrolled} + c.\text{waitlist} \\
\]

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Extension 1

• Add an attribute for students
  – Unique ID numbers
  – This requires a new signature

• Add student transcripts

• Add prerequisite structure for courses
New Relations

sig Id {}

abstract sig Student extends Person {
    id: one Id,
    transcript: set Course
}

sig Graduate, Undergrad extends Student {}

sig Instructor in Person {}

sig Course {
    taughtby: one Instructor,
    enrolled: some Student,
    waitlist: set Student,
    prerequisites: set Course
}
New Constraints

• Each Student is identified by one unique ID
  – Exactly one ID per Student
    already enforced by multiplicities
  – No two distinct students have the same ID
    has to be specified as a fact

• A student’s transcript contains a course only if it contains the course’s prerequisites

• A course does not have itself as a prerequisite

• Realism: there exists a course with prerequisites and with students enrolled
Academia Constraints

**fact** {
...

  -- A student’s transcript contains a course only if it contains the course’s prerequisites
  all s: Student |
    s.transcript.prerequisites in s.transcript

  -- A course does not have itself as a prerequisite
  all c: Course | c !in c.prerequisites
}

**run** {
...

  -- there is a course with prerequisites and enrolled students
  some c: Course |
    some c.prerequisites and some c.enrolled
}
Academia Constraints

fact {
...

-- A student’s transcript contains a course only if it contains the course’s prerequisites
all s: Student |
  s.transcript.prerequisites in s.transcript

-- There are no cycles in the prerequisite dependencies
all c: Course | c !in c.^prerequisites
}

run {
...

-- there is a course with prerequisites and enrolled students
some c: Course |
  some c.prerequisites and some c.enrolled
}
• Students can only wait to be in a course for which they already have the prerequisites

```assert AllWaitsHavePrereqs {
  all s: Student |
  (waitlist.s).prerequisites in s.transcript
}
Exercises

• Load academia-2.als

• With realism conditions enabled, do any instances exist in the default scopes?
  – Manipulate the scopes as necessary to obtain an instance under the realism conditions

• By looking at various sample instances, do you consider the model to be underconstrained in any way?
Counter-example

Analyzing AllWaitsHavePrereqs ...

Counterexample found:

Signatures:
  Id = \{Id0, Id1, Id2\}
  Course = \{C0, C1\}
  Person = \{U, G0, G1\}
  Faculty = \{
  Student = \{U, G0, G1\}
  Undergrad = \{U\}
  Graduate = \{G0, G1\}
  Instructor = \{G0, G1\}

Relations:
  taughtby = \{(C0, G0), (C1, G0)\}
  enrolled = \{(C0, U), (C1, G1)\}
  waitlist = \{(C1, U)\}
  prerequisites = \{(C1, C0)\}
  transcript = \{(G1, C0)\}
  id = \{(U, Id0), (G0, Id2), (G1, Id1)\}

---

\textbf{U waits for the course C1 and C0 is a prerequisite for C1 but U does not have C0}

\textbf{Where is (U,C0)?}
New constraint

• Old Assertion \textit{AllWaistsHavePrereqs}
  Students can \textit{wait} only for those courses for which they already have the prerequisites

• Old Fact
  Students can \textit{have} a course only if they already have the prerequisites

• New Fact
  Students can \textit{have, wait for or take} a course only if they already have the prerequisites
New constraint

- New Fact: A student can **have, wait for or take** a course only if they already have the prerequisites

\[
\text{all } s:\text{ Student} \mid
\quad (\text{waitlist.s.prerequisites} + \\
\quad \text{enrolled.s.prerequisites} + \\
\quad \text{s.transcript.prerequisites})
\quad \text{in } s\text{.transcript}
\]

\[
\text{all } s:\text{ Student} \mid
\quad (\text{waitlist.s} + \text{enrolled.s} + \text{s.transcript})\text{.prerequisites } \text{in } s\text{.transcript}
\]
Extension 2

• Add Departments, with
  – Instructors
  – Courses
  – Required courses
  – Student majors

• Add Faculty-Grad student relationships
  – Advisor
  – Thesis committee
Department Relations

• Each *instructor* is in a single *department*
  – Each *department* has at least one *instructor*

• Each *department* has some *courses*
  – *Courses* are in a single *department*

• Each *student* has a single *department* as his/her *major*
Faculty-Student Relations

• A *graduate* student has exactly one *faculty* member as an *advisor*

• *Faculty* members serve on *graduate* students’ *committees*
New Relations

```ml
sig Faculty extends Person {
    incommittee: set Graduate
}

abstract sig Student extends Person {
    major: one Department
}

sig Graduate extends Student {
    advisor: one Faculty
}

sig Instructor in Person {
    department: one Department
}

sig Department {
    course: some Course,
    required: some course
}
```

------------------------ Facts ------------------------

-- Each department has at least one instructor
all d: Department | some department.d

-- Each course is in a single department
all c: Course | one course.c
New Constraints

• Advisors are on their advisees’ committees
• Students are advised by faculty in their major
• Only faculty can teach required courses
• Faculty members only teach courses in their department
• Required courses for a major are a subset of the courses in that major
• Students must be enrolled in at least one course from their major
Exercise

• Express as an Alloy fact each of the new constraints in the previous slide
Advisors are on their advisees’ committees

------------------ Signatures and Fields ------------------

abstract sig Person {} 

sig Faculty extends Person {
    incommittee: set Graduate
}

abstract sig Student extends Person {
    id: one Id,
    transcript: set Course,
    major: one Department
}

sig Undergrad extends Student {}

sig Graduate extends Student {
    advisor: one Faculty
}

sig Instructor in Person {
    department: one Department
}

sig Course {
    taughtby: one Instructor,
    enrolled: some Student,
    waitlist: set Student,
    prerequisites: set Course
}

sig Id {}

sig Department {
    courses: some Course,
    required: some Course
}
Students are advised by faculty in their major

---------- Signatures and Fields ----------

abstract sig Person {}  
sig Faculty extends Person {  
    incommittee: set Graduate  
}
abstract sig Student extends Person {  
    id: one Id,  
    transcript: set Course,  
    major: one Department  
}
sig Undergrad extends Student {}  
sig Graduate extends Student {  
    advisor: one Faculty  
}
sig Instructor in Person {  
    department: one Department  
}
sig Course {  
    taughtby: one Instructor,  
    enrolled: some Student,  
    waitlist: set Student,  
    prerequisites: set Course  
}
sig Id {}  
sig Department {  
    courses: some Course,  
    required: some Course  
}
Required courses for a major are a subset of the courses in that major

------------- Signatures and Fields -------------

abstract sig Person {}
sig Faculty extends Person {
incommittee: set Graduate
}
abstract sig Student extends Person {
id: one Id,
transcript: set Course,
major: one Department
}
sig Undergrad extends Student {}
sig Graduate extends Student {
advisor: one Faculty
}
sig Instructor in Person {
department: one Department
}
sig Course {
taughtby: one Instructor,
enrolled: some Student,
waitlist: set Student,
prerequisites: set Course
}
sig Id {}
sig Department {
courses: some Course,
required: some Course
}
Only faculty teach required courses

----------------------------- Signatures and Fields -----------------------------

abstract sig Person {}
sig Faculty extends Person {
  incommittee: set Graduate
}
abstract sig Student extends Person {
  id: one Id,
  transcript: set Course,
  major: one Department
}
sig Undergrad extends Student {}
sig Graduate extends Student {
  advisor: one Faculty
}
sig Instructor in Person {
  department: one Department
}
sig Course {
  taughtby: one Instructor,
  enrolled: some Student,
  waitlist: set Student,
  prerequisites: set Course
}
sig Id {}
sig Department {
  courses: some Course,
  required: some Course
}
Faculty members only teach courses in their department

------------------ Signatures and Fields ------------------

abstract sig Person {}

sig Faculty extends Person {
incommittee: set Graduate
}

abstract sig Student extends Person {
id: one Id,
transcript: set Course,
major: one Department
}

sig Undergrad extends Student {}

sig Graduate extends Student {
advisor: one Faculty
}

sig Instructor in Person {
department: one Department
}

sig Course {
  taughtby: one Instructor,
enrolled: some Student,
  waitlist: set Student,
  prerequisites: set Course
}

sig Id {}

sig Department {
courses: some Course,
required: some Course
}
Students must be enrolled in at least one course from their major

------------------ Signatures and Fields ------------------

```pseudocode
abstract sig Person {}
sig Faculty extends Person {
incommittee: set Graduate
}
abstract sig Student extends Person {
id: one Id,
transcript: set Course,
major: one Department
}
sig Undergrad extends Student {}
sig Graduate extends Student {
advisor: one Faculty
}
sig Instructor in Person {
department: one Department
}
sig Course {
taughtby: one Instructor,
enrolled: some Student,
waitlist: set Student,
prerequisites: set Course
}
sig Id {}
sig Department {
courses: some Course,
required: some Course
}
```

------------------
There are at least two departments and some required courses

------------------- Signatures and Fields -------------------

abstract sig Person {}

sig Faculty extends Person {
  incommittee: set Graduate
}

abstract sig Student extends Person {
  id: one Id,
  transcript: set Course,
  major: one Department
}

sig Undergrad extends Student {}

sig Graduate extends Student {
  advisor: one Faculty
}

sig Instructor in Person {
  department: one Department
}

sig Course {
  taughtby: one Instructor,
  enrolled: some Student,
  waitlist: set Student,
  prerequisites: set Course
}

sig Id {}

sig Department {
  courses: some Course,
  required: some Course
}
A student’s committee members are faculty in his/her major

------------------- Signatures and Fields -------------------

abstract sig Person {}

sig Faculty extends Person {
  incommittee: set Graduate
}

abstract sig Student extends Person {
  id: one Id,
  transcript: set Course,
  major: one Department
}

sig Undergrad extends Student {}

sig Graduate extends Student {
  advisor: one Faculty
}

sig Instructor in Person {
  department: one Department
}

sig Course {
  taughtby: one Instructor,
  enrolled: some Student,
  waitlist: set Student,
  prerequisites: set Course
}

sig Id {}

sig Department {
  courses: some Course,
  required: some Course
}
Assertions

• Realism constraints: There are at least two departments and some required courses

• Assertion: A student’s committee members are faculty in his/her major
Exercises

• Load academia-3.als
• With realism conditions enabled, do any instances exist in the default scopes?
• Manipulate the scopes as necessary to obtain an instance under the realism conditions
  – This requires some thought since constraints may interact in subtle ways
  – For example, adding a department requires at least one faculty member for that department
• Can you think of any more questions about the model?
  – Formulate them as assertions and see if the properties are already enforced by the constraints