1. (20 points) Show that NP is closed under union and concatenation.

2. (20 points) Call graphs $G$ and $H$ isomorphic if the nodes of $G$ may be reordered so that it is identical to $H$. Let

$$ISO = \{ \langle G, H \rangle \mid G \text{ and } H \text{ are isomorphic graphs} \}$$

Show that $ISO \in NP$.

3. (20 points) Let

$$MODEXP = \{ \langle a, b, c, p \rangle \mid a, b, c \text{ and } p \text{ are positive binary integers such that } a^b \equiv c \text{ (mod } p) \}$$

Show that $MODEXP \in P$. (Note that the most obvious algorithm doesn’t run in polynomial time. Hint: try it first where $b$ is a power of 2.)

4. (20 points) Show that if $P = NP$, then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete.

5. (20 points) We generally believe that $PATH$ is not NP-complete. Explain the reason behind this belief. Show that proving $PATH$ is not NP-complete would prove $P \neq NP$. 

6. (20 points) Show that NP is closed under union and concatenation.