1. (20 points) Show that a language $A$ is decidable if and only if $A \leq_m 0^*1^*$, in which $L_1 \leq_m L_2$ denotes “$L_1$ is mapping reducible to $L_2$.”

2. (15 points) Let $J = \{w \mid$ either $w = 0x$ for some $x \in A_{TM}$, or $w = 1y$ for some $y \in \overline{A_{TM}}\}$. Show that neither $J$ nor $J$ is Turing-recognizable.

3. (15 points) Consider the problem of determining whether a PDA accepts some string of the form $\{ww \mid w \in \{0,1\}^*\}$. Use the computation history method to show that this problem is undecidable.

4. (30 points) Say that a variable $A$ in CFG $G$ is necessary if it appears in every derivation of some string $w \in L(G)$. Let $NECESSARY_{CFG} = \{\langle G, A \rangle \mid A$ is a necessary variable in $G\}$.
   (a) Show that $NECESSARY_{CFG}$ is Turing-recognizable.
   (b) Show that $NECESSARY_{CFG}$ is undecidable.

5. (20 points) Show that $ALL_{DFA} = \{\langle M \rangle \mid M$ is a DFA and $L(M) = \Sigma^*\}$ is in $P$. 