1. (15 points) Let \( \text{INFINITE}_{\text{PDA}} = \{ \langle M \rangle \mid M \text{ is a PDA and } \mathcal{L}(M) \text{ is an infinite language} \} \). Show that \( \text{INFINITE}_{\text{PDA}} \) is decidable.

2. (15 points) Let \( A = \{ \langle R, S \rangle \mid R \text{ and } S \text{ are regular expressions and } \mathcal{L}(R) \subseteq \mathcal{L}(S) \} \). Show that \( A \) is decidable.

3. (15 points) Let \( C = \{ \langle G, x \rangle \mid G \text{ is a CFG and } x \text{ is a substring of some } y \in \mathcal{L}(G) \} \). Show that \( C \) is decidable. 
   (Hint: An elegant solution to this problem uses the decider for \( E_{\text{CFG}} \), the language encoding the emptiness problem for CFGs.)

4. (15 points) Show that the set of all infinite binary sequences is uncountable. An infinite binary sequence is an unending sequence of 0s and 1s.

5. (20 points) Show that \( E_{\text{DFAs}} = \{ \langle A, B \rangle \mid A, B \text{ are DFAs, and } \mathcal{L}(A) = \mathcal{L}(B) \} \) is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works.

6. (20 points) Let \( A \) be a Turing-recognizable language consisting of descriptions of Turing Machines, \( \{ \langle M_1 \rangle, \langle M_2 \rangle, \ldots \} \), where every \( M_i \) is a decider. Show that some decidable language \( D \) is not decided by any decider \( M_i \) whose description appears in \( A \).
   (Hint: You may find it helpful to consider an enumerator for \( A \).)