Here is a summary of rules for negating statements. Refer to the “appendices” which I handed out at the beginning of the term, and to section 4 of the class notes for more details.

- The negation of “A or B” is “not(A) and not(B).”
- The negation of “A and B” is “not(A) or not(B).”
- The negation of “For every x, P(x)” is “There exists x such that not(P(x)).”
- The negation of “There exists an x such that P(x)” is “For every x, not(P(x)).”
- The negation of “A implies B” is “A and not(B).” But note, many statements with implications have implicit universal quantifiers. For example, consider the statement: “If L and M are distinct lines with non-empty intersection, then the intersection of L and M consists of one point.” This actually means: For every pair of lines L and M, if L and M are distinct and have non-empty intersection, then the intersection of L and M consists of one point.” Therefore the negation uses both the rule for negation of sentences with universal quantifiers, and the rule for negation of implications: “There exists a pair of lines L and M such that L and M are distinct and have non-empty intersection, and the intersection does not consist of one point.” This can be rephrased as: “There exists a pair of lines L and M such that L and M are distinct and have at least two points in their intersection.”

1. Form the negation of each of the following sentences; no quantifier should appear within the scope of a not(), and the negation should be expressed in natural English.
   (a) Tonight I will go to a restaurant for dinner or to a movie.
   (b) Tonight I will go to a restaurant for dinner and to a movie.
   (c) If today is Tuesday, I have missed a deadline.
   (d) For all lines L, L has at least two points.
   (e) For every line L and every plane P, if L is not a subset of P, then L ∩ P has at most one point.
   (f) (In the following statement, f is understood to be a function from a set A to a set B. The statement is the definition of f being onto.) For every b ∈ B there exists an a ∈ A such that f(a) = b.
(g) (Same context as the previous part. The statement is the definition of \( f \) being 1-to-1.) For every \( a_1, a_2 \in A \) if \( a_1 \neq a_2 \), then \( f(a_1) \neq f(a_2) \).

2. Same instructions as for the previous problem. Watch out for implicit universal quantifiers.
   (a) If \( x \) is a real number, then \( \sqrt{x^2} = |x| \).
   (b) If \( x \) is a natural number and \( x \) is not a perfect square, then \( \sqrt{x} \) is irrational.
   (c) If \( n \) is a natural number, then there exists a natural number \( N \) such \( N > n \).
   (d) If \( L \) and \( M \) are distinct lines, then either \( L \) and \( M \) do not intersect, or their intersection contains exactly one point.

3. Form the contrapositive of the following implications.
   (a) For every line \( L \) and every plane \( \mathbb{P} \), if \( L \) is not a subset of \( \mathbb{P} \), then \( L \cap \mathbb{P} \) has at most one point.
   (b) For every line \( L \) and every plane \( \mathbb{P} \), if \( L \) is not a subset of \( \mathbb{P} \), then \( L \cap \mathbb{P} \) has at most one point.
   (c) (In the following statement, \( f \) is understood to be a function from a set \( A \) to a set \( B \). The statement is the definition of \( f \) being onto.) For every \( b \in B \) there exists an \( a \in A \) such that \( f(a) = b \).
   (d) (Same context as the previous part. The statement is the definition of \( f \) being 1-to-1.) For every \( a_1, a_2 \in A \) if \( a_1 \neq a_2 \), then \( f(a_1) \neq f(a_2) \).

4. Form a/the converse of the implications in the previous exercise.

5. Prove Lemma 2.3 in the class notes.

6. Prove Theorem 3.8 in the class notes.