**Math 34, Second Midterm Exam – Solutions**
**Fall, 2003**

(1) *Solve the initial value problem:*

$$y' = 2y(1-y), \quad y(0) = 1/2.$$

This is the logistics equation (page 76, equation (7)) with $r = 2$ and $K = 1$. This exercise also appeared on the first exam.

For the solution, see pages 78-79, or check your class notes.

(2) (a) *What is meant by* linear independence *of a set* $\{f_1, f_2, \ldots, f_s\}$ *of functions?*

This question is from the second review sheet, where an appropriate answer is also given: The set $\{f_1, f_2, \ldots, f_s\}$ of functions is *linearly independent* if the only solution $(c_1, c_2, \ldots, c_s)$ to the equation

$$c_1 f_1 + c_2 f_2 + \cdots + c_s f_s = 0$$

is $c_1 = c_2 = \cdots = c_s = 0$.

Remark: Two possible approaches to preparing for a question like this are: (i) memorize the answer or (ii) understand the concept so thoroughly that you can put it in your own words. For most people, approach (i) is a much safer bet, and, in fact, is a necessary first step towards understanding the concept.

(b) *What is the relation between any particular solution and the general solution of the following differential equation:*

$$2y'' - 5y' - 3y = 7t + t^2 e^{-t/2}$$

Either of the following answers would be sufficient:

*Answer 1:* The general solution of the equation is the sum of any particular solution plus the general solution to the associated homogeneous equation

$$2y'' - 5y' - 3y = 0.$$

*Answer 2:* Let $Y(t)$ be a particular solution to the equation. Then the general solution to the equation is

$$Y(t) + c_1 e^{-t/2} + c_2 e^{3t}.$$

(3) *Solve the initial value problem:*

$$16y'' - 8y' + 145y = 0, \quad y(0) = 1, \quad y'(0) = -2.$$

The solutions to the characteristic equation

$$16\,r^2 - 8\,r\ + 145 = 0$$

are $r = 1/4 \pm 3i$. Therefore, a fundamental family of solutions to the equation is $\{e^{t/4}\cos(t), e^{t/4}\sin(t)\}$, and the general solution is

$$f(t) = c_1\,e^{t/4}\cos(3t) + c_2\,e^{t/4}\sin(3t).$$

We still have to find appropriate constants $c_1$ and $c_2$ in order to satisfy the initial conditions. Setting $t = 0$, we get $f(0) = c_1$, so $c_1 = 1$. The derivative of $f(t)$ is

$$\frac{c_1\,e^{\frac{t}{4}}\ \cos(3\,t)}{4} + 3\,c_2\,e^{\frac{t}{4}}\ \cos(3\,t) - 3\,c_1\,e^{\frac{t}{4}}\ \sin(3\,t) + \frac{c_2\,e^{\frac{t}{4}}\ \sin(3\,t)}{4}.$$

Setting $t = 0$, we get $f'(0) = (1/4)c_1 + 3c_2$. Therefore, $(1/4)c_1 + 3c_2 = -2$. Solving, we get $c_1 = 1, c_2 = -3/4$. Thus, the solution to the initial value problem is

$$f(t) = e^{t/4}\cos(3t) - (3/4)e^{t/4}\sin(3t).$$

(4) *For each of the following inhomogeneous 2nd order linear differential equations, propose a particular solution, with undetermined coefficients.* Do not proceed to solve for the undertermined coefficients.

(a) $2y'' - 5y' - 3y = 7t + t^2e^{-t/2}$
(b) $2y'' - 2y' + 5y = 7t + t^2e^{-t/2}$

The proposed particular solution for the method of undetermined coefficients depends almost exclusively on the RHS of the inhomogeneous DE, but has to be modified slightly if the RHS involves a solution to the associated homogenous equation.

For part (a), since $e^{-t/2}$ is a solution to the homogenous equation, the proposed solution has the form:

$$(A + Bt) + t(C + Dt + Et^2)e^{-t/2}.$$

For part (b), the RHS does not involve a solution to the associated homogenous equation, so the proposed solution has the form:

$$(A + Bt) + (C + Dt + Et^2)e^{-t/2}.$$

Remark: It is a misconception that you can find a solution with fewer summands. For example, for part (b), many people proposed a solution of the form

$$(Bt) + (Et^2)e^{-t/2}.$$

In fact, this just doesn't work.