(1) Solve the initial value problem:
\[ y' = 2y(1 - y), \quad y(0) = 1/2. \]

This is the logistics equation (page 76, equation (7)) with \( r = 2 \) and \( K = 1 \). This exercise also appeared on the first exam.

For the solution, see pages 78-79, or check your class notes.

(2) (a) What is meant by linear independence of a set \( \{f_1, f_2, \ldots, f_s\} \) of functions?

This question is from the second review sheet, where an appropriate answer is also given: The set \( \{f_1, f_2, \ldots, f_s\} \) of functions is linearly independent if the only solution \((c_1, c_2, \ldots, c_s)\) to the equation
\[ c_1 f_1 + c_2 f_2 + \cdots + c_s f_s = 0 \]
is \( c_1 = c_2 = \cdots = c_s = 0 \).

Remark: Two possible approaches to preparing for a question like this are: (i) memorize the answer or (ii) understand the concept so thoroughly that you can put it in your own words. For most people, approach (i) is a much safer bet, and, in fact, is a necessary first step towards understanding the concept.

(b) What is the relation between any particular solution and the general solution of the following differential equation:
\[ 2y'' - 5y' - 3y = 7t + t^2e^{-t/2} \]

Either of the following answers would be sufficient:

Answer 1: The general solution of the equation is the sum of any particular solution plus the general solution to the associated homogeneous equation
\[ 2y'' - 5y' - 3y = 0. \]

Answer 2: Let \( Y(t) \) be a particular solution to the equation. Then the general solution to the equation is
\[ Y(t) + c_1 e^{-t/2} + c_2 e^{3t}. \]
(3) Solve the initial value problem:

\[ 16y'' - 8y' + 145y = 0, \quad y(0) = 1, \quad y'(0) = -2. \]

The solutions to the characteristic equation

\[ 16r^2 - 8r + 145 = 0 \]

are \( r = 1/4 \pm 3i \). Therefore, a fundamental family of solutions to the equation is \( \{ e^{t/4} \cos(t), e^{t/4} \sin(t) \} \), and the general solution is

\[ f(t) = c_1 e^{t/4} \cos(3t) + c_2 e^{t/4} \sin(3t). \]

We still have to find appropriate constants \( c_1 \) and \( c_2 \) in order to satisfy the initial conditions. Setting \( t = 0 \), we get \( f(0) = c_1 \), so \( c_1 = 1 \). The derivative of \( f(t) \) is

\[ \frac{c_1 e^{t/4} \cos(3t)}{4} + \frac{3c_2 e^{t/4} \cos(3t) - 3c_1 e^{t/4} \sin(3t)}{4} + \frac{c_2 e^{t/4} \sin(3t)}{4}. \]

Setting \( t = 0 \), we get \( f'(0) = (1/4)c_1 + 3c_2 \). Therefore, \( (1/4)c_1 + 3c_2 = -2 \). Solving, we get \( c_1 = 1, c_2 = -3/4 \). Thus, the solution to the initial value problem is

\[ f(t) = e^{t/4} \cos(3t) - (3/4)e^{t/4} \sin(3t). \]

(4) For each of the following inhomogeneous 2nd order linear differential equations, propose a particular solution, with undetermined coefficients. Do not proceed to solve for the undetermined coefficients.

(a) \( 2y'' - 5y' - 3y = 7t + t^2 e^{-t/2} \)
(b) \( 2y'' - 2y' + 5y = 7t + t^2 e^{-t/2} \)

The proposed particular solution for the method of undetermined coefficients depends almost exclusively on the RHS of the inhomogeneous DE, but has to be modified slightly if the RHS involves a solution to the associated homogenous equation.

For part (a), since \( e^{-t/2} \) is a solution to the homogenous equation, the proposed solution has the form:

\( (A + Bt) + t(C + Dt + Et^2) e^{-t/2}. \)

For part (b), the RHS does not involve a solution to the associated homogenous equation, so the proposed solution has the form:

\( (A + Bt) + (C + Dt + Et^2) e^{-t/2}. \)
Remark: It is a misconception that you can find a solution with fewer summands. For example, for part (b), many people proposed a solution of the form

\((Bt) + (Et^2)e^{-t/2}\).

In fact, this just doesn’t work.