Evaluate the integrals in exercises 1–3. If you use a substitution, say explicitly what substitution you are using. If you do integration by parts, say explicitly what are the parts.

(1) \( \int xe^{3x} \, dx \).

(2) \( \int \frac{1}{x\sqrt{4 - x^2}} \, dx \).

(3) \( \int \sin^2(x) \cos^3(x) \, dx \).

(4) The rational function

\[
f(x) = \frac{x}{(x^2 + 4)(x - 1)^2}
\]

has the partial fractions expansion

\[
-\frac{3x}{25(x^2 + 4)} + \frac{3}{25(x - 1)} - \frac{8}{25(x^2 + 4)} + \frac{1}{5(x - 1)^2}.
\]

Compute \( \int f(x) \, dx \).

(5) Consider the integral

\[
\int_1^6 \frac{1}{x^2} \sqrt{1 + 4/x^6} \, dx.
\]

The fourth derivative of the integrand

\[
f(x) = \frac{1}{x^2} \sqrt{1 + 4/x^6}
\]

is decreasing (and positive) on the interval \([1, 6]\), and the value of the fourth derivative at \(x = 1\) is a little less than 3363. How many subintervals (an even number!) do you have to use in Simpson’s rule to obtain an approximation to the integral with an error less than \(10^{-5}\)?

Please don’t just write down an answer, even though you use your calculator for this one. Write the formulas you are using and indicate what values you are assigning to the parameters in the formulas.
(6) Determine whether the integral
\[ \int_{1}^{\infty} \frac{1}{x^2} \sqrt{1 + \frac{4}{x^6}} \, dx \]
converges or diverges, by an appropriate comparison. (*Hint:* There
is an “obvious” function to compare with, but the inequality goes
the wrong way! You have to adjust the comparison function a bit to
get an inequality that goes the right way.)

Some formulas.

(1) The following formulas come from the double angle formulas:
\[
\cos^2(a) = \frac{1}{2}(\cos(2a) + 1), \quad \text{and} \quad \sin^2(a) = \frac{1}{2}(1 - \cos(2a)).
\]

(2)
\[
\int \sec(x) \, dx = \ln |\sec(x) + \tan(x)| + c,
\]
\[
\int \csc(x) \, dx = -\ln |\csc(x) + \cot(x)| + c.
\]

(3) These come from the angle addition formulas:
\[
\sin(a) \cos(b) = \frac{1}{2} (\sin(a - b) + \sin(a + b))
\]
\[
\sin(a) \sin(b) = \frac{1}{2} (\cos(a - b) - \cos(a + b))
\]
\[
\cos(a) \cos(b) = \frac{1}{2} (\cos(a - b) + \cos(a + b))
\]

(4) The trapezoid rule approximation for
\[
\int_{a}^{b} f(x) \, dx
\]
is
\[
\frac{\Delta(x)}{2} \left( y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n \right),
\]
where \( \Delta(x) = (b - a)/n \), and \( y_i = f(x_i) \). The error estimate for the
trapezoid rule is
\[
|\text{error}| \leq \frac{K(b - a)^3}{12 n^2},
\]
where \( K \) is an upper bound for \( |f^{(2)}(x)| \) on \([a, b]\).

(5) Simpson’s rule approximation for
\[
\int_{a}^{b} f(x) \, dx
\]
is
\[
\frac{\Delta(x)}{3} \left( y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n \right),
\]
where \( \Delta(x) = (b - a)/n \), \( y_i = f(x_i) \), and \( n \) is even. The error
estimate for Simpson’s rule is
\[
|\text{error}| \leq \frac{K(b - a)^5}{180 n^4},
\]
where \( K \) is an upper bound for \( |f^{(4)}(x)| \) on \([a, b]\).