Engineering Calculus – 22m35
Final Exam
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Instructions: This exam has 7 problems. Do all the problems. Show your work for full credit.

1. **(a)** Give the definition of the derivative of a function.

(b) Compute the derivative of $f(x) = \frac{1}{\sqrt{x}}$ directly from the definition. Show your steps. Check your answer by differentiating $f(x)$ using appropriate rules of differentiation.

(c) Find the tangent line to $y = \frac{1}{\sqrt{x}}$ at x = 2.

2. **(a)** State the Fundamental Theorem of Calculus; don't forget to include the hypotheses.

(b) Find the derivative of $F(x) = \int_1^x \sin(t^2)\cos(t)\,dt$.

3. Graph the function $y = \dfrac{x^2}{1+x}$ using first and second derivative information and paying particular attention to local maxima and minima, points of inflection, intervals on which the function is increasing or decreasing, intervals on which the function is concave up or concave down, and asymptotes, if any. Check your work by checking the consistency of your data.

4. **(a)** Consider the equation $8 + 8x + x^3 = 0$. Observe that the function $f(x) = 8 + 8x + x^3$ is negative when $x = -1$ and positive when x = 0. Therefore there is a solution to the equation between -1 and 0.

Find this solution by using Newton's method (with some reasonable initial guess). Show your successive approximations to the solution.

(b) Use first derivative information to show that the equation $f(x) = 8 + 8x + x^3 = 0$ does not have more than one real solution. Hint: Think of the Mean Value Theorem or Rolle's Lemma.

5. Consider a pyramid with a square base of side a and with height h. Show by integration that the volume of the pyramid is $(1/3)a^2h$.

6. Calculate the integrals. Don't forget to check your answers by differentiation:

(a) $\displaystyle\int x\sqrt{2x+3}\,dx$

(b) $\displaystyle\int_1^3 \frac{x}{x^2-4}\,dx$

(c) $\displaystyle\int x^5\sqrt{x^2-1}\,dx$

7. Find the dimensions of the cylinder of maximum volume which can be inscribed in a sphere of radius R.