Math 16, Second Midterm Review Questions
April, 2003

1. Work through the review sheet for the first midterm, as well as the first midterm.
2. Graph the following functions, using first and second derivative information. Find out on what intervals the functions are increasing/decreasing and on what intervals the functions are concave up/down. Find all local maxima and minima and all inflection points. Also consider possible horizontal or vertical asymptotes.
   (a) $f(x) = x^2 e^{-x}$.
   (b) $f(x) = 3x^3 - 2x^2 + 5x - 4$.
   (c) $f(x) = (5x - 2)/(3x - 10)$
3. A piece of wire of length L is bent into a rectangle with sides a and b. Find the dimensions a and b which maximize the area of the rectangle.
4. Find the point on the line $x + y = 1$ which is closest to the point $(1, 3)$. Hint: Minimize the square of the distance. The square of the distance is $(x - 1)^2 + (y - 3)^2$. Use the equation of the line to eliminate one of the variables.
5. Describe the geometric idea of Newton's method and illustrate your description with a picture. What is the formula for the next approximant (x_{n+1}) in terms of the current approximant (x_n) in Newton's method? Derive this formula.
6. Use Newton's method to (begin to) compute the cube root of 5. Choose an appropriate function $f(x)$ such that the solution to $f(x) = 0$ is the cube root of 5. Start with an appropriate guess (x_0) and compute 3 steps of Newton's method "by hand" (i.e. using a pocket calculator).
7. Describe how you can find all solutions of $10251 - 3969x + 181x^2 + x^3 = 0$, without actually carrying out the computations. How much of the computation can be done "by hand", i.e., without using a computer? Can you find out how many solutions there are and choose appropriate starting guesses for applying Newton's method in order to find the solutions?
8. What is meant by a differential equation? What is meant by a solution to a differential equation? What is meant by a slope field, and what does a slope field have to do with a differential equation? What is meant by "the general solution" to a differential equation? What is meant by a "particular solution" to a differential equation? What is an "inital value problem".

9. Write down the differential equation for exponential growth. What is the general solution to this differential equation? A bacterial population at low densities satisfies an exponential growth law. It's initial value is 1000 bacteria and its doubling time is 3 hours. Find the population at any time t (t in hours).
10. Write down the differential equation form of Newton's law of cooling. What is the general solution to this differential equation? A hot potato is taken out of the microwave oven at 99 degrees C and placed on your dinner plate in a room at 22 degrees C. You plan to eat it when it reaches 40 degrees C. Inspired by your calculus class, you stick a digital thermometer into the potato and notice that after 3 minutes it has reached 85 degrees C. At what time will you be able to eat the potato?
11. A pollutant enters a lake at a rate of 200 g/day. Due to the flow of water through the lake, the pollutant in the lake is removed from the lake at a rate of 5% of its current amount per day. Write down the differential equation for this pollution model. Find the general solution. If $2000g$ of pollutant is present in the lake at time zero, what is the amount of pollutant in the lake at any time t (t measured in days)?
12. What is the logistics differential equation and in what sense it is the "next step in population modeling after the equation of exponential growth"? What is the general solution to the logistics differential equation?
13. Graph the logistics function
$$f(t) = \frac{1000}{1 + 40e^{-.1t}}$$
What are its asymptotes? At what value of t is $f'(t)$ maximum? What is the corresponding value of $f(t)$?