Math 16, Homework 3

Exercises 1, 2, and 6 are presentation exercises.

For exercises 1-2, find the exact derivative of the function $f(x)$ by the method we have considered in class; that is, consider $\dfrac{f(x+h)-f(x)}{h}$ and manipulate the expression until the limit as $h \to 0$ can be evaluated by inspection. Do these calculations by hand, to practice the appropriate algebra. Later in this assignment, you will do some similar computations using Mathematica.

1. $f(x) = \dfrac{1}{x^2}$.

2. $f(x) = \dfrac{1}{\sqrt{x}}$.

3. Find the tangent line to $y = \sqrt{x}$ at $(2, \sqrt{2})$.

4. Find the derivative of $f(x) = x - 3x^2$. Over what interval(s) is $f(x)$ increasing and over what intervals is $f(x)$ decreasing? Find the exact value of x where $f(x)$ is maximum and find the maximum value of the function.
5. For several (positive) values of a of your choice, calculate (numerically and approximately) the limit $\lim\limits_{h\to 0} \dfrac{a^h - 1}{h}$, and compare with $\ln(a)$.

6. A bacterial population satisfies the growth law
$$P(t) = P_0 e^{kt},$$
for a certain value of k, where t is measured in hours, and P_0 denotes the population at time $t = 0$. It is observed that the population doubles in 3.3 hours. Use this observation to find k. Find the rate of growth of the bacterial culture for all t.

7. Define a function $\ell(x)$ for positive x by the formula $\ell(x) = \lim\limits_{h\to 0} \dfrac{x^h - 1}{h}$. Pretend that you haven't been told that $\ell(x) = \ln(x)$. Show that the function $\ell(x)$ has the basic property of a logarithm function: $\ell(ab) = \ell(a) + \ell(b)$. Hint:
$$(ab)^h - 1 = a^h b^h - 1 = (a^h b^h - a^h) + (a^h - 1).$$

MATHEMATICA exercises.

Work through the third Mathematica tutorial, Mathematica as a symbolic calculator, which you can get from the course web page.

Then do the following exercises (to be handed in). First open the template notebook which you have prepared. Save a copy of it called symbolicDerivative.nb. Do your work in this saved copy. Change the title to "Symbolic calculation of the derivative."

Follow the example at the end of the tutorial (the section called symbolic calculation of some derivative) to calculate the derivatives of the following functions exactly:

1. $f(x) = x^7$.
2. $f(x) = 1/x^4$.