

Math 16, Homework 4

1. For each of the following pairs of functions, find $f \circ g$ and $g \circ f$:

(a) $f(x) = 2x + 3, \quad g(x) = 5x + 4.$

(b) $f(x) = \frac{x}{1+x}, \quad g(x) = \frac{1+x}{x}.$

(c) $f(x) = x^2 + 1, \quad g(x) = \frac{1+x}{x}.$

2. Find the n -fold iteration of the function $f(x) = 125 + .6x$.

3. Find the inverses of each of the following functions:

(a) $f(x) = \frac{x}{1+x}.$

(b) $g(x) = \frac{2+x}{x}.$

4. The following plot of the function $y = f(x) = \frac{1}{1+x^2}$ shows that the the range of the function is $R = \{y : 0 < y \leq 1\}$. The function is *not* 1-to-1. Find a suitable domain D so that the *restriction* of the function to this domain, $f : D \rightarrow R$ is one-to-one and onto, and find the inverse of the function.

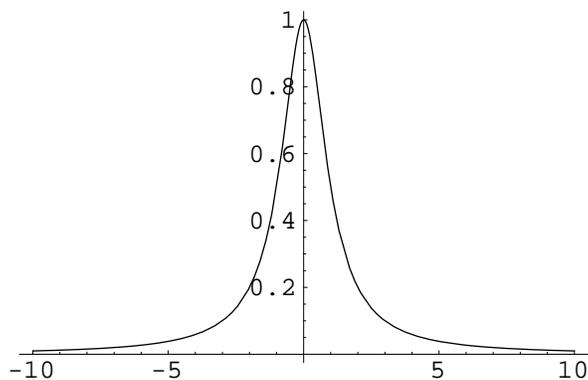


FIGURE 0.1. $y = \frac{1}{1+x^2}$

5. The following plot of the function $y = f(x) = \frac{x}{1+x^2}$ shows that the the range of the function is $R = \{y : -.5 < y \leq .5\}$. The maximum value .5 is attained at $x = 1$, and the minimum value $-.5$ is attained at $x = -1$. The function is *not* one-to-one. However, there are three intervals such that the restriction of the function to each of these intervals is one-to-one. Find

these three intervals. Find (or guess) the range of the function restricted to each of the three intervals. Can you find the inverse of the function restricted to each of the three intervals?

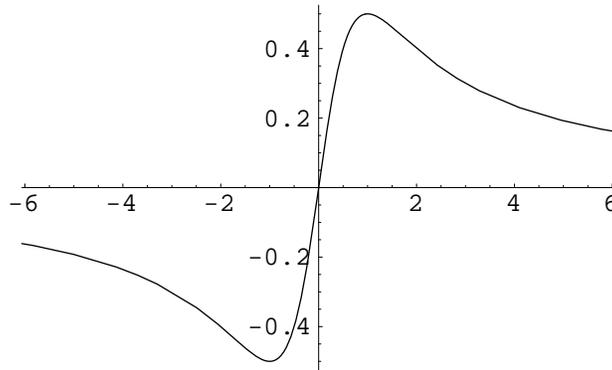


FIGURE 0.2. $y = \frac{x}{1+x^2}$

6. Textbook exercise 1.5.5.
7. Textbook exercise 1.5.6.
8. The growth of a certain population is governed by the law $P(t) = (1.14)^t P(0)$, where t is measured in years. Determine the amount of time required for the population to double.
9. The decay of a certain radioactive substance is governed by the law $C(t) = (.87)^t C(0)$. Determine the number of years required for the amount of the substance to be reduced by one half (the half-life).
10. The decay of a certain radioactive substance is governed by the law $C(t) = k^t C(0)$. If the half-life of the substance is 3000 years, find k .