1. Show that given 52 integers, there exist two of them whose sum or difference is divisible by 100.

2. (a) How many ways are there to distribute 3 identical oranges, 5 identical candy canes, and one lump of coal to 3 (distinguishable) holiday stockings?
   (b) What if the candy canes are of five distinct flavors?

3. (a) Count the permutations of the multiset \( \{2a, 2b, 2c\} \).
   (b) Count the circular permutations of the multiset \( \{2a, 2b, 2c\} \), using Polya-Burnside theory. That is, count the number of orbits of the cyclic group \( \mathbb{Z}_6 \) acting on multiset permutations.

4. How many integer solutions are there to:
   (a) \( x_1 + x_2 + x_3 + x_4 = 15, \ x_i \geq 0? \)
   (b) \( x_1 + x_2 + x_3 + x_4 \leq 15, \ x_i \geq 0? \)
   (c) \( x_1 + x_2 + x_3 + x_4 = 15, \ x_i \geq 0, \ x_2 \geq 2, \ x_4 \leq 3? \)

5. Give generating function models for each of the parts of problem 4; that is, replace 15 by a variable \( n \), and write the generating function for the number of solutions to the equation or inequality.

6. (a) Give the generating function for distributions of \( r \) identical rice krispies into 17 distinguishable rice krispy boxes. (This is the same as the generating function for solutions to \( x_1 + x_2 + \cdots + x_{17} = r \).)
   (b) Give the generating function for distributions of \( r \) identical rice krispies into 17 indistinguishable rice krispy boxes. (This is the same as the generating function for solutions to \( x_1 + x_2 + \cdots + x_{17} = r \), where the order of the \( x_i \)'s doesn’t matter. Since the order doesn’t matter, arrange the \( x_i \)'s in decreasing order. So you are actually counting partitions of \( r \) with no more than 17 parts!)
   (c) Give the generating function for distributions of \( r \) identical rice krispies into \( r \) indistinguishable rice krispy boxes. (Now there is no longer any restriction on the number of parts in the partitions.)

Remark: The generating function for partitions of \( n \) with parts in a particular subset \( S \) of the natural numbers is
\[
\prod_{j \in S} \frac{1}{1 - x^j};
\]
in particular the generating function for all partitions is
\[
\prod_{j=1}^{\infty} \frac{1}{1 - x^j};
\]