Mathematics 118 Review for Midterm I October, 2005

Definitions you should know:

- **1.** Analytic function.
- 2. Convergent sequence, convergent series, absolutely convergent series.
- **3.** Uniformly convergent series of functions.
- 4. Open set, closed set, connected set.
- 5. Isolated singularity.
- 6. Residue of a function at an isolated singularity.
- 7. Removable singularity, pole, essential singularity.
- 8. Path integral $\int_{\gamma} f(z) dz$.

Theorems you should be able to (state and) prove:

- 1. Derive Cauchy-Riemann equations.
- 2. Cauchy integral theorem from Green's theorem and Cauchy-Riemann equations.
- 3. Cauchy integral formula from Cauchy integral theorem.
- 4. Existence of power series expansion, using Cauchy integral formula and expansion of geometric series.
- 5. Liouville's theorem from Cauchy estimates.

Theorems/formulas you should be able to state/reproduce:

- 1. Morera's theorem.
- 2. Taylor formula for coefficients of a power series.
- **3.** Cauchy integral formulas for higher derivatives of an analytic function, or equivalently for the coefficients of a power series.
- 4. Cauchy estimates for derivatives. (Easily derived from previous item!)

Be able to do typical computations from the homework.