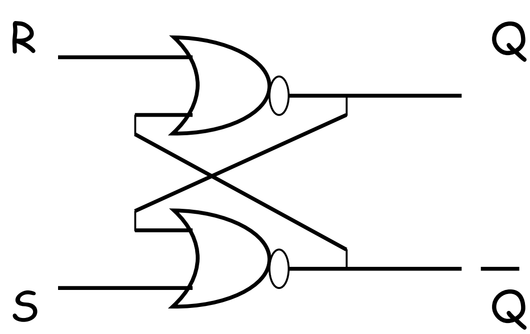
Sequential Circuits

The output depends not only on the current inputs, but also on the past values of the inputs. This is how a digital circuit remembers data. Let us see how a single bit is stored.

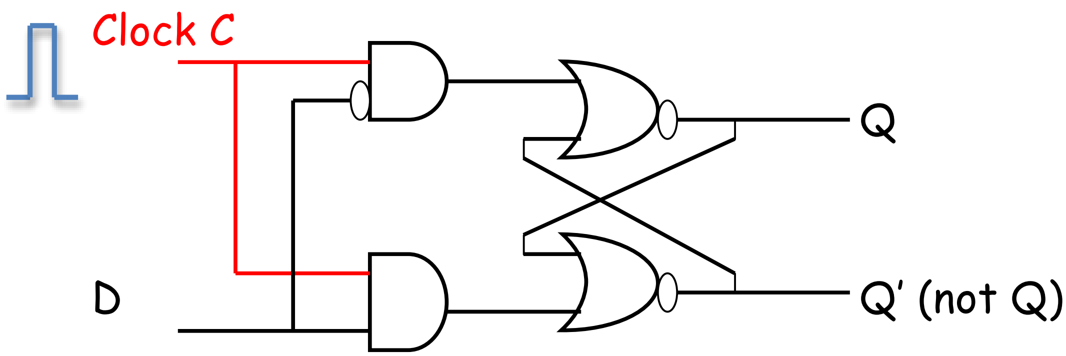


An SR Latch

R = Reset, S= Set

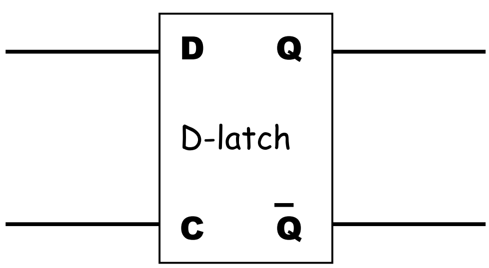
S	R	Q	$\overline{Q}$	Comment
0	0	0/1	1/0	Old state continues
1	0	1	0	Set state
0	1	0	1	Reset state
1	1	0	0	Illegal inputs

A clocked D-latch



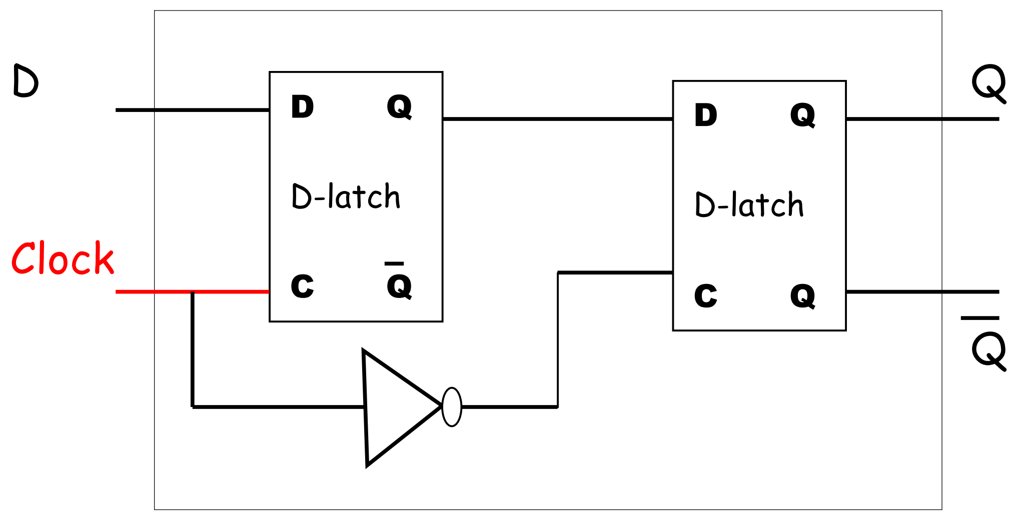
Clock is the enabler. If C=0, Q remains unchanged.

When C=1, then Q acquires the value of D. We will use it as a building block of sequential circuits.

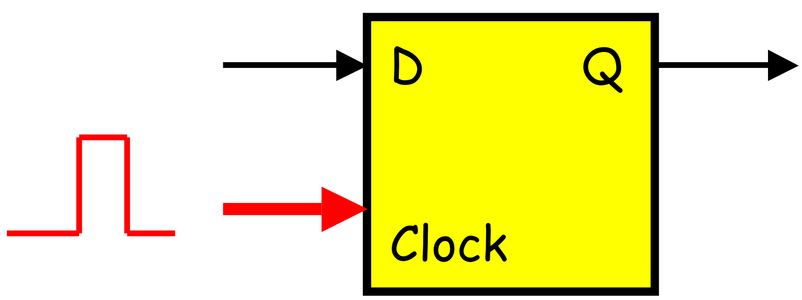


There are some shortcomings of this simple circuit. An edge-triggered circuit (or a master-slave circuit) solves this problem

Master-Slave D flip-flop



Internal details shown above

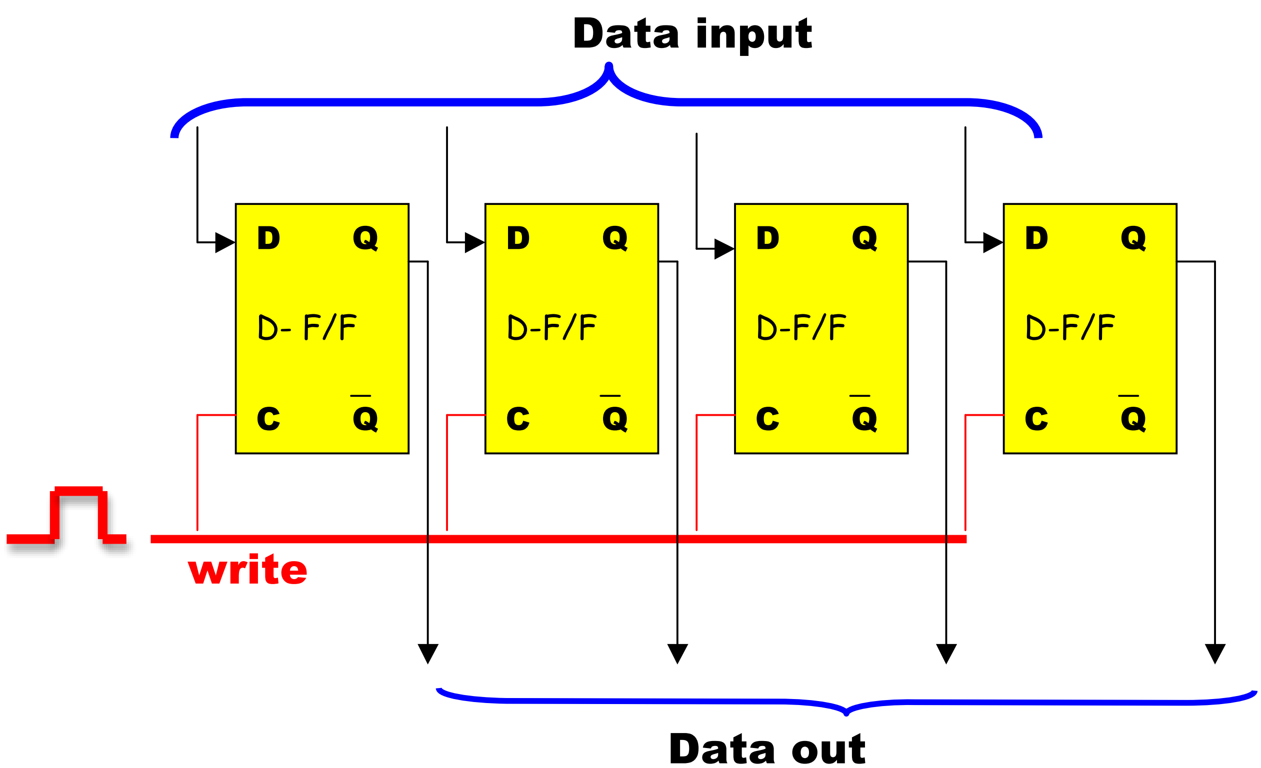


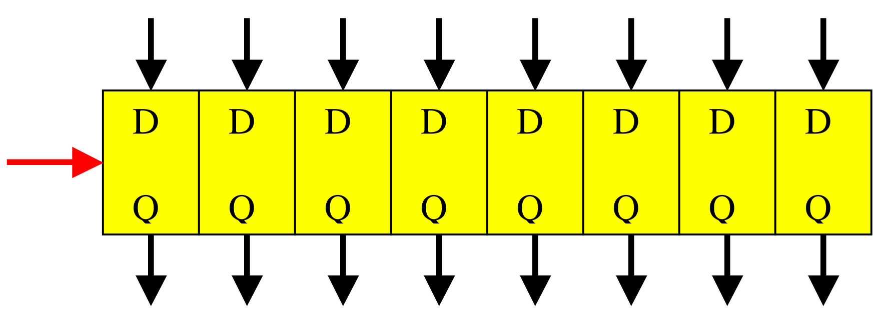
Clock pulse Abstract view

The output Q acquires the value of the input D, only when one complete clock pulse is applied to the clock input.

Register

A 8-bit register is an array of 8 D-flip-flops.

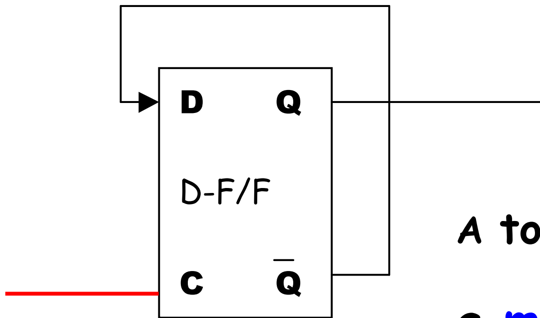




Abstract view of a register

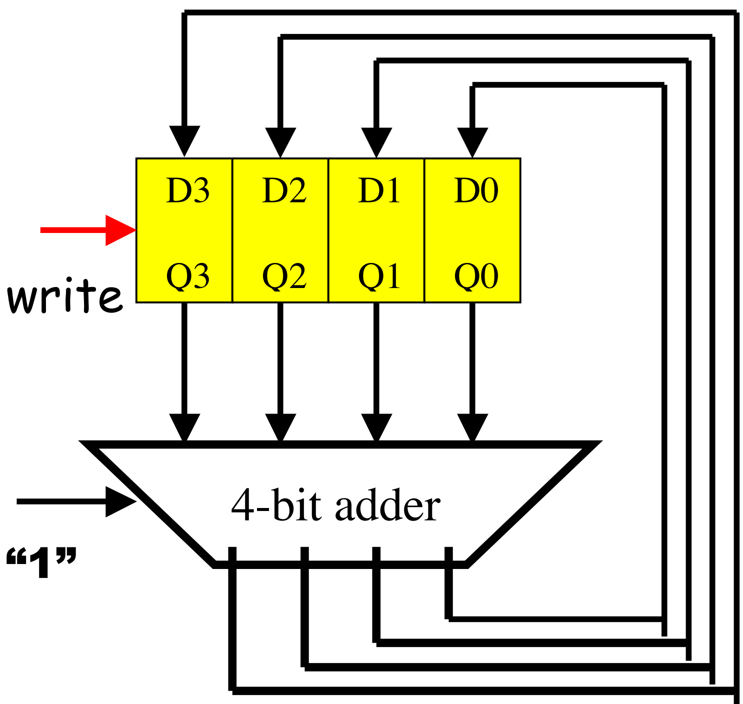
Binary counter

Counts 0, 1, 2, 3, …



A toggle flip-flop (T) is

a **modulo-2 counter**



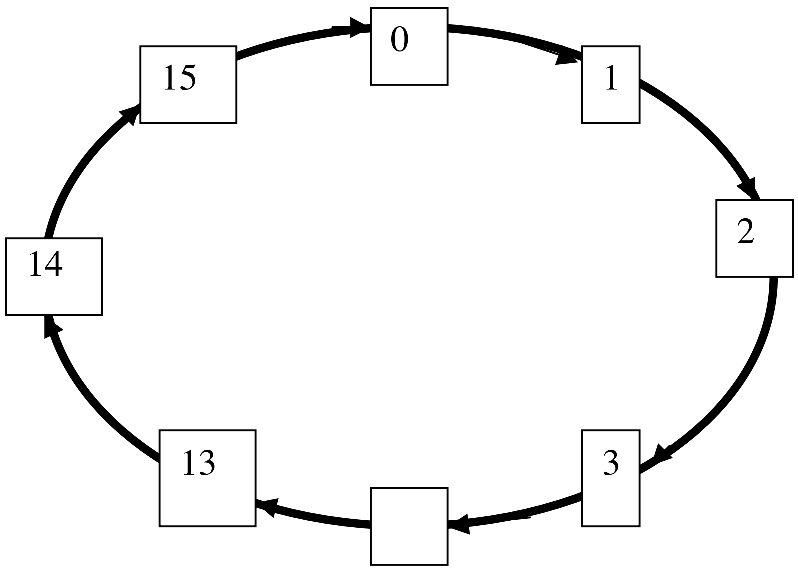
A 4-bit counter

(mod-16 counter)

Observe how Q3 Q2 Q1 Q0 change when pulses are applied to the clock input

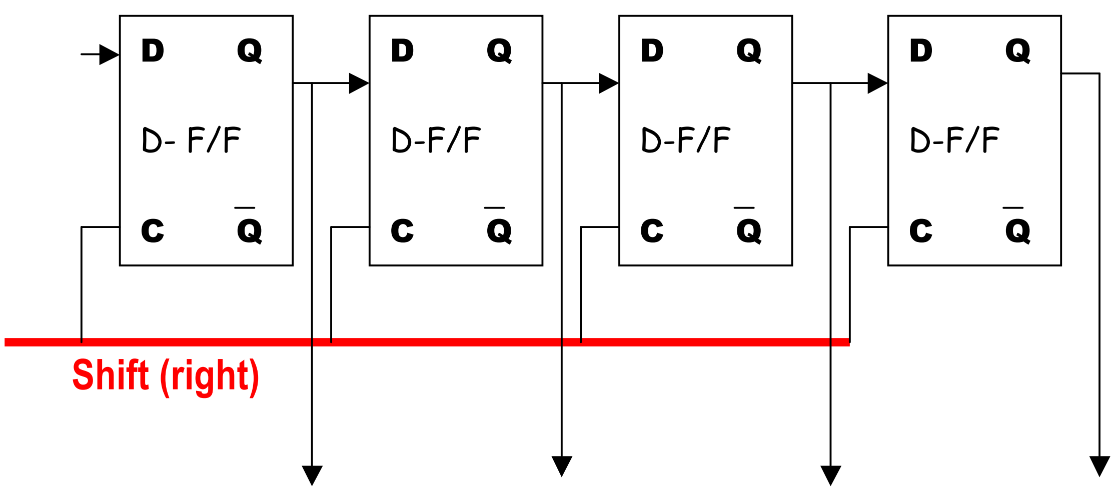
State diagram of a 4-bit counter

Here state = Q3Q2Q1Q0



Recall that the program counter is a 32-bit counter

A shift register

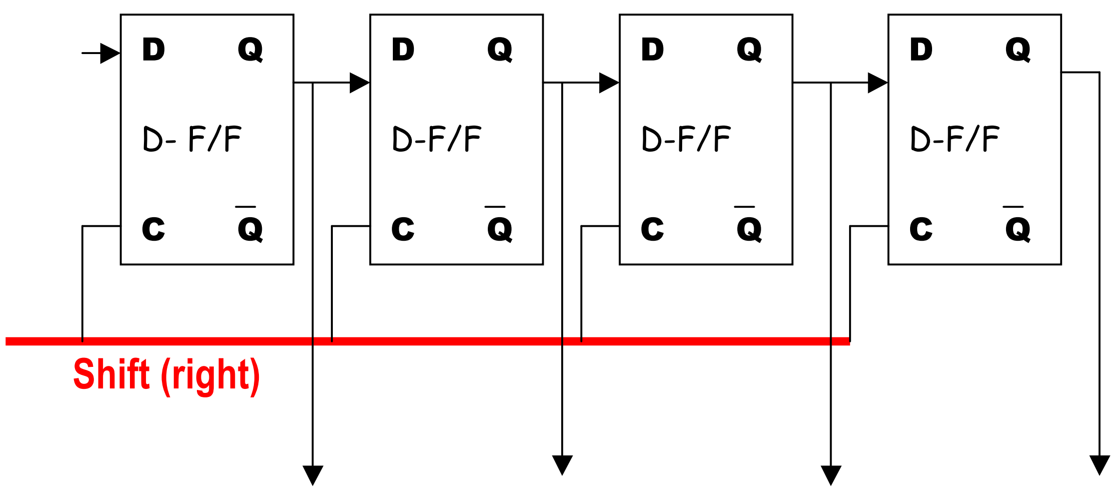


With each pulse

The Building Blocks

A shift register

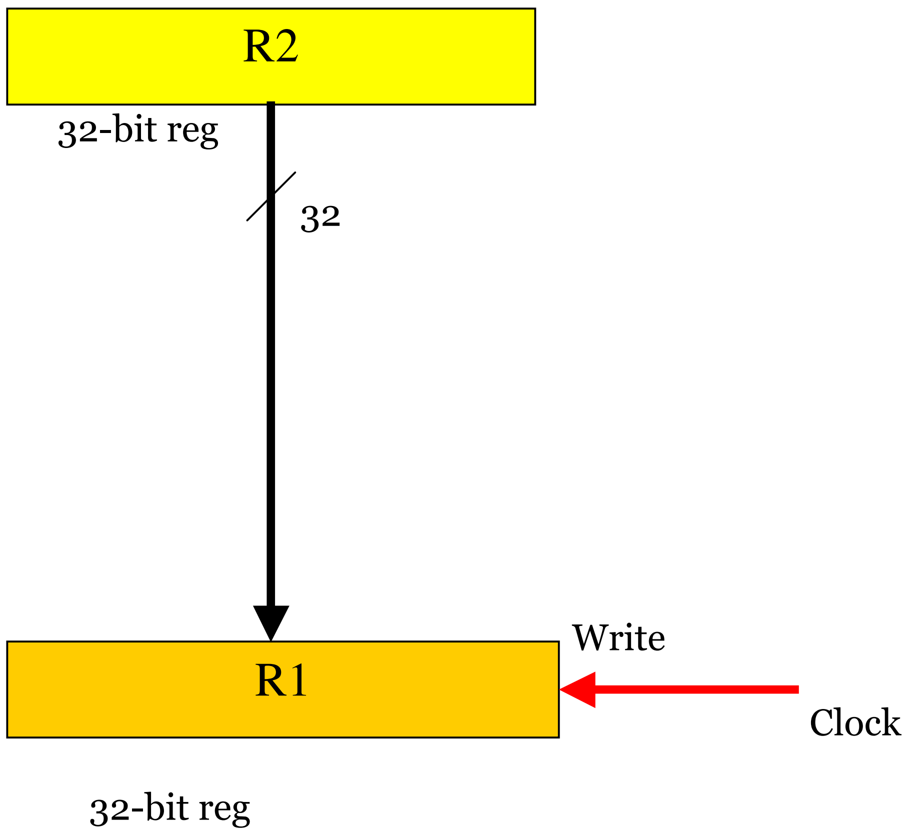
Review how a D flip-flop works



With each clock pulse on the shift line, data moves one place to the right.

Executing r1:= r2

How to implement a simple register transfer r1:= r2?

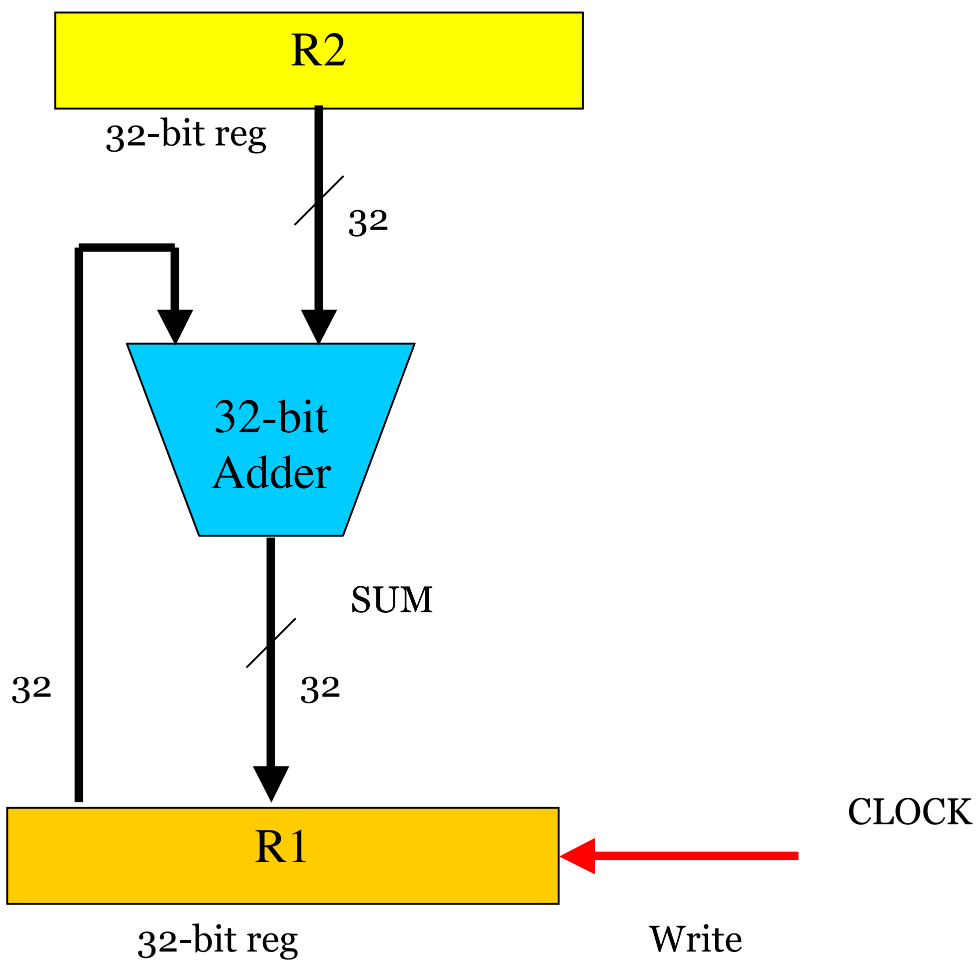


It takes one clock pulse to complete the operation.

Q. How to swap the contents of R1, R2?

Computer Arithmetic in ALU

Adding two registers: Executing r1:= r1+r2



It requires only one clock pulse to complete the operation.

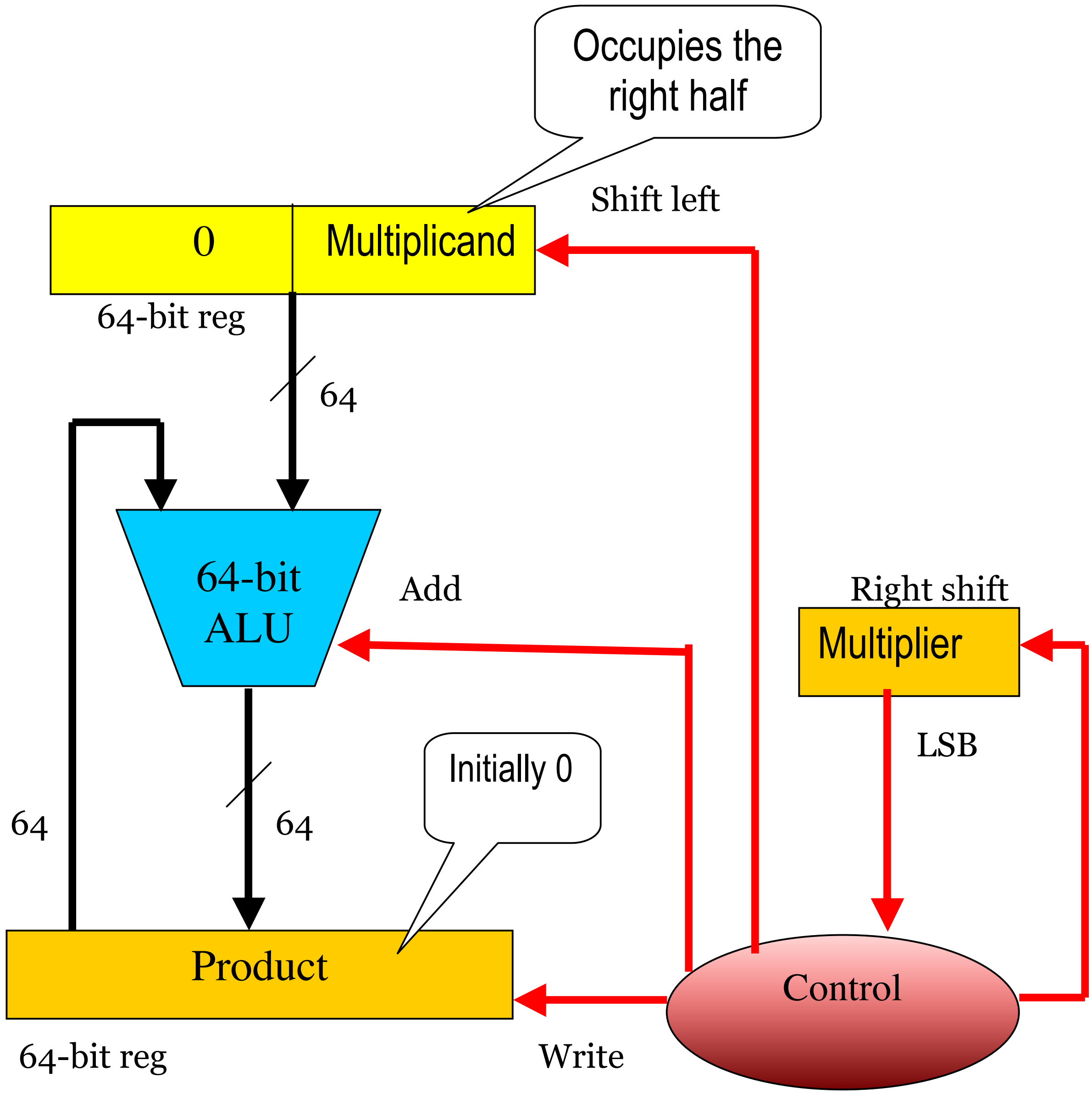
Hardware Multiplication

By now, you know all the **building blocks**.

Multiplicand				1	0	0	1
Multiplier				1	0	1	0
				0	0	0	0
			1	0	0	1	0
		0	0	0	0	0	0
	1	0	0	1	0	0	0
Product	1	0	1	1	0	1	0

The basic operations are ADD and SHIFT. Now let us see how it is implemented by hardware.

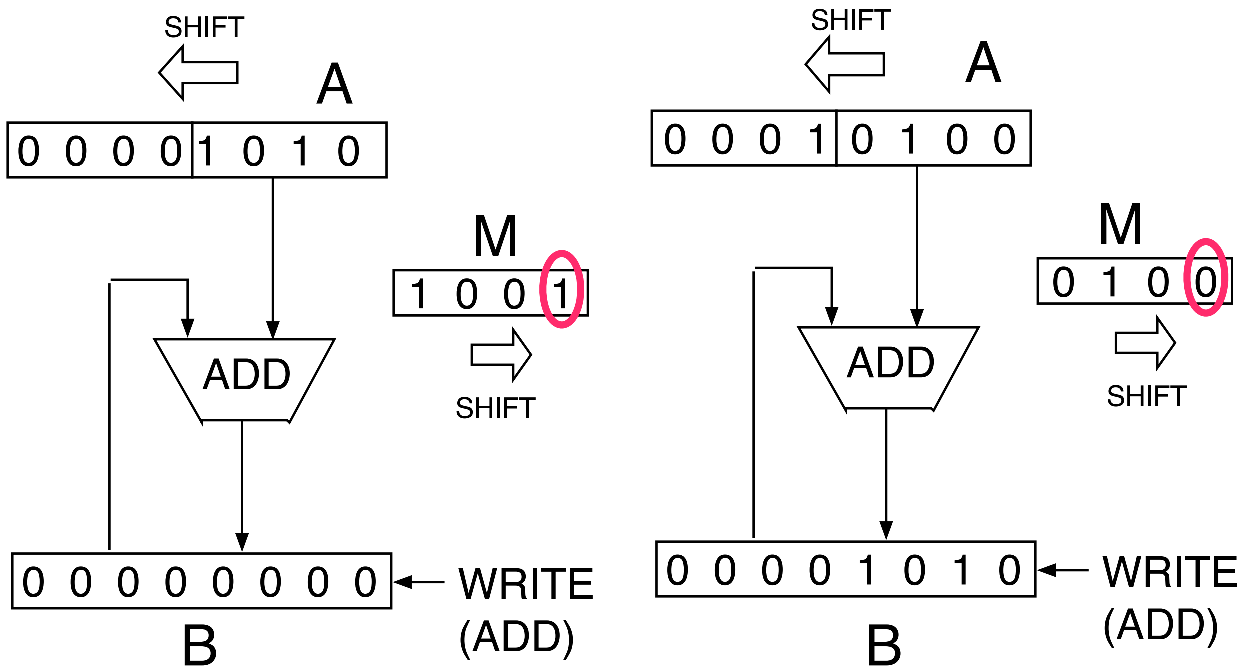
A Hardware Multiplier



If LSB of Multiplier = 1 then *add* else *skip*;

Shift left multiplicand & shift right multiplier

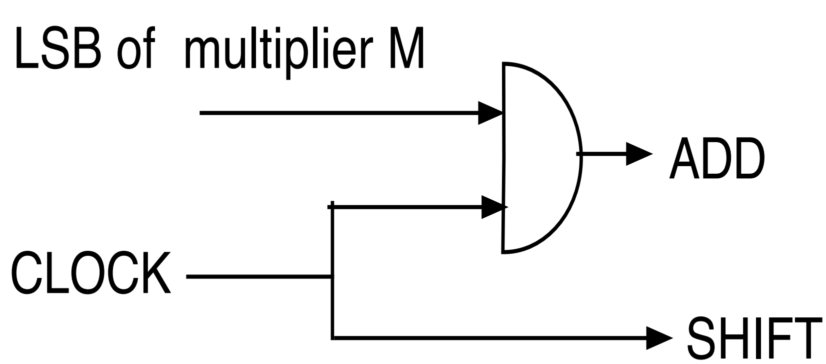
How to implement the control unit?



if LSB (M) = 1 then ADD, SHIFT LEFT A, SHIFT RIGHT M

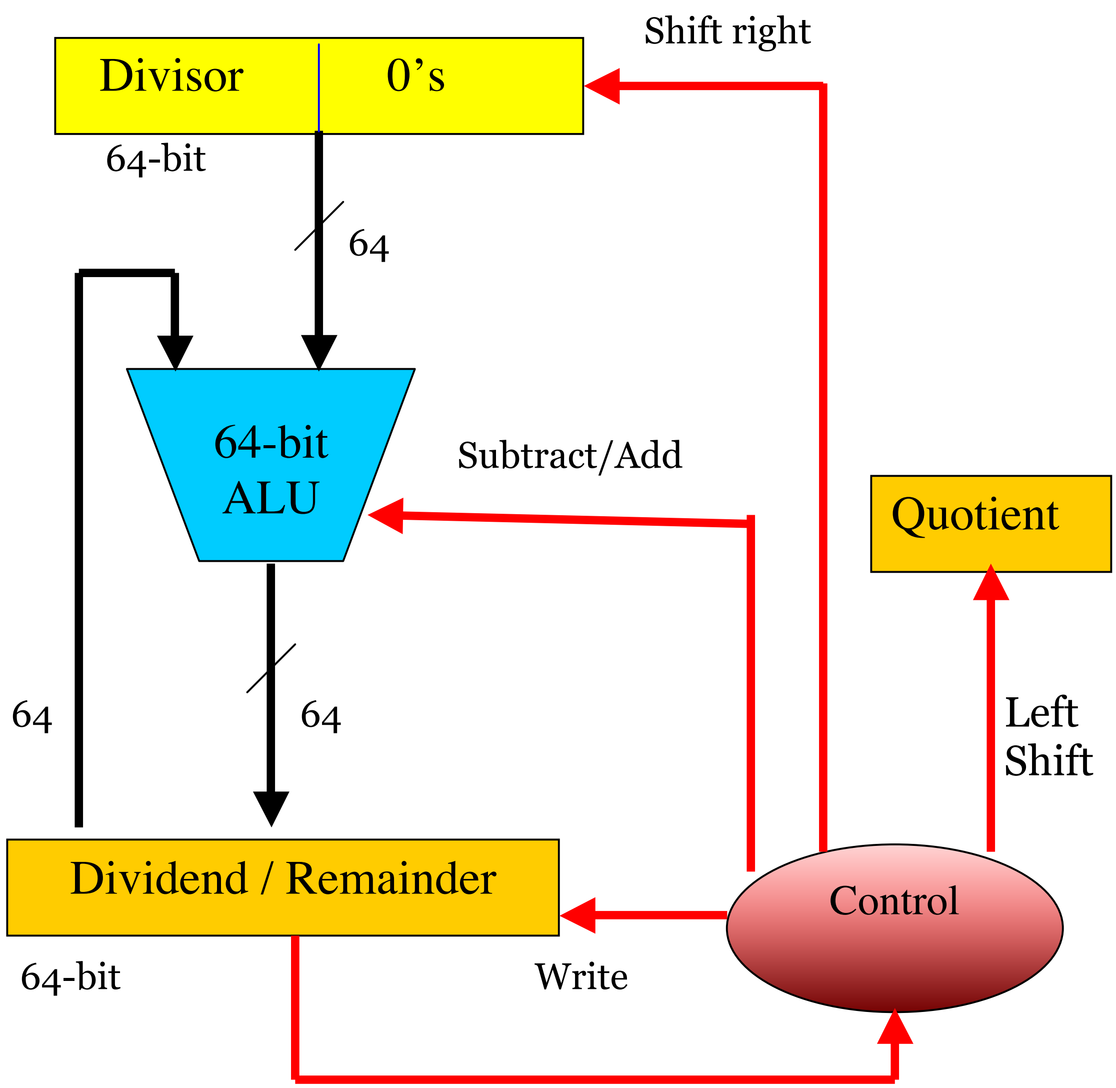
else SHIFT LEFT A, SHIFT RIGHT M

The Control Unit for the Multiplier



Division

The restoring division algorithm uses the simple idea from the elementary school days. It uses subtraction and shift. Here is an implementation by hardware



How does it work?

Floating point Representation of Numbers

Floating Point representation is useful for representing a number in a wide range: **very small** to **very large**. It is widely used in the scientific world. Consider, the following FP representation of a number

Exponent E significand F (also called ***mantissa***)

+/-	x x x x	y y y y y y y y y y y y

Sign bit

It means (+/-) $\mathbf{1}.yyyyyyyyyyyy \times 2^{xxxx}$

(The 1 is implied)

IEEE 754 single-precision (32 bits)

s	xxxxxxxx	yyyyyyyyyyyyyyyyyyyyyyy
1	8	23 bits

Single precision

Largest = 1. 1 1 1 … x 2^{+127} ≈ 2 x 10^{+38}

Smallest = 1.000 … x 2^{-128} ≈ 1 x 10^{-38}

These can be positive and negative, depending on s.

(But there are exceptions too)

IEEE 754 double precision (64 bits)

S	exponent	significand
1	11 bits	52 bits

Largest = 1. 1 1 1… x 2^{+1023}

Smallest = 1.000… X 2^{-1024}

Overflow and **underflow** in FP

An **overflow** occurs when the number if too large to fit in the frame. An **underflow** occurs when the number is too small to fit in the given frame.

How do we represent zero?

IEEE standards committee solved this by making **zero** a special case: if every bit is zero (the sign bit being irrelevant), then the number is considered zero.

Then how do we represent 1.0?

Then how do we represent 1.0?

It should have been 1.0×2^0 (same as 0)! The way out of this is that the interpretation of the exponent bits is not straightforward. The exponent of a single-precision float is **"shift-127" encoded** (**biased representation**), meaning that the actual exponent is (xxxxxxx minus 127). So thankfully, we can get an exponent of zero by storing 127.

Exponent = 11111111 (i.e. 255) means 255-127 = 128
Exponent = 01111111 (i.e. 127) means 127-127 = 0
Exponent = 00000001 (i.e. 1) means 1-127 = -126

More on Biased Representation

The consequence of shift-127

Exponent = 00000000 (reserved for 0) can no more be used to represent the smallest number. We forego something at the lower end of the spectrum of representable exponents, (which could be 2^{-127}). That said, it seems wise, to give up the smallest exponent instead of giving up the ability to represent 1 or zero!

More special cases

Zero is not the only "special case" float. There are also representations for positive and negative infinity, and for a not-a-number (NaN) value, for results that do not make sense (for example, non-real numbers, or the result of an operation like infinity times zero). How do these work? A number is infinite if every bit of the exponent is 1 (yes, we lose another one), and is NaN if every bit of the exponent is 1 plus any mantissa bits are 1. The sign bit still distinguishes +/-inf and +/-NaN. Here are a few sample floating point representations:

Exponent	**Mantissa**	**Object**
0	0	Zero
0	Nonzero	Denormalized number*
1-254	Anything	+/- FP number
255	0	+ / - infinity
255	Nonzero	NaN like 0/0 or 0x inf

* Any non-zero number that is smaller than the smallest normal number is a denormalized number. The production of a denormal is sometimes called gradual underflow because it allows a calculation to lose precision slowly when the result is small.

Floating point operations in MIPS

32 separate single precision FP registers in MIPS

f0, f1, f2, … f31,

Can also be used as 16 double precision registers

f0, f2, f4, f30 (f0 means f0,f1 f2 means f2,f3)

These reside in a **coprocessor** C1 in the same package

Operations supported

add.**s**	$f2, $f4, $f6	# f2 = f4 + f6 (single precision)
add.**d**	$f2, $f4, $f6	# f2 = f4 + f6 (double precision)

(Also subtract, multiply, divide format are similar)

lwc1	$f1, 100($s2)	# f1 = M [s2 + 100] (32-bit load)
mtc1	$t0, $f0	# f0 = t0 (move to coprocessor 1)
mfc1	$t1, $f1	# t1 = f1 (move from coprocessor 1)

Sample program

Evaluation of a Polynomial $a.x^2 + b.x + c$

Pseudo-instruction

```
        # $f0 --- x
        # $f2 --- sum of terms
        . . . . .

        # Evaluate the quadratic
        l.s      $f2,a           # sum = a
        mul.s    $f2,$f2,$f0     # sum = ax

        l.s      $f4,b           # get b
        add.s    $f2,$f2,$f4     # sum = ax + b
        mul.s    $f2,$f2,$f0     # sum = (ax+b)x = ax^2 + bx

        l.s      $f4,c           # get c
        add.s    $f2,$f2,$f4     # sum = ax^2 + bx + c
        . . . . . .

        .data
a:      .float  1.0
b:      .float  1.0
c:      .float  1.0
```

Floating Point Addition

Example using decimal

$A = 9.999 \times 10^{1}$, $B = 1.610 \times 10^{-1}$, $A+B = ?$

Step 1. Align the smaller exponent with the larger one.

$B = 0.0161 \times 10^{1} = 0.016 \times 10^{1}$ (round off)

Step 2. Add significands

$9.999 + 0.016 = 10.015$, so $A+B = 10.015 \times 10^{1}$

Step 3. Normalize

$A+B = 1.0015 \times 10^{2}$

Step 4. Round off

$A+B = 1.002 \times 10^{2}$

Exercise. Add 0.5 and –0.4375 in binary.

Floating Point Multiplication

Example using decimal

$A = 1.110 \times 10^{10}$, $B = 9.200 \times 10^{-5}$ $A \times B = ?$

Step 1. Exponent of $A \times B = 10 + (-5) = 5$

Step 2. Multiply significands

$1.110 \times 9.200 = 10.212000$

Step 3. Normalize the product

$10.212 \times 10^{5} = 1.0212 \times 10^{6}$

Step 4. Round off

$A \times B = 1.021 \times 10^{6}$

Step 5. Decide the sign of $A \times B$ $(+ \times + = +)$

So, $A \times B = + 1.021 \times 10^{6}$