22C: 166 Distributed Systems and Algorithms

Homework 2, Total points = 60

(Worth 6% of your total grade)

Assigned 2/17/15 due 2/24/15 in class

Please submit typewritten solutions unless you are proud of your handwriting.

Question 1 (20+10+10=40 points): Consider a connected network (V, E), where each vertex $v \in V$ represents an *unbounded clock,* and E denotes the set of edges. All clocks are ticking at the same rate, and displaying the same value. Assume that an adversary perturbs one or more of these clock values. The following program synchronizes the clock values in a bounded number of steps following a perturbation:

{program for clock i}
define $c[i]$: integer {non-negative integer representing value of clock i}
{ $N(i)$ denotes the set of neighbors of clock i }
do true $\rightarrow$ $c[i] := 1 + \max\{c[j] : j \in N(i) \cup \{i\}\}$ **od**

Consider a synchronous model, where all clocks simultaneously execute the above action with each clock tick, and the action takes zero time to complete.

(a) Use a well-founded set and an appropriate variant function to show that the above algorithm will enable the clock display identical readings in a bounded number of steps.

(b) What is the maximum number of rounds needed by the clocks to be synchronized?

(c) Will the above algorithm work for bounded size clocks too? Briefly justify your answer.

Question 2 (10 points): In a completely connected network of $n > 1$ processes, each process i has an integer variable $x[i]$. Initially $\forall i : x[i] = 0$. Let $N(i)$ denote the set of neighbors of process i. The system is asynchronous and each process i executes the following program:

do $\forall j \in N(i) : x[i] \le x[j] \rightarrow x[i] + 1$
[] $\exists j \in N(i) : x[i] > x[j] \rightarrow skip$
od

Prove that the safety property $\forall i, j : | x[i] - x[j] | \le 1$ holds. You need to demonstrate that if the property holds before the execution of an action, then it holds after the action is executed.

Question 3 (10 points): Consider running Ricart and Agrawala's algorithm on a ring of five processes 0-4. What are the maximum and the minimum number of processes that will enter their critical sections? Briefly justify your answer.