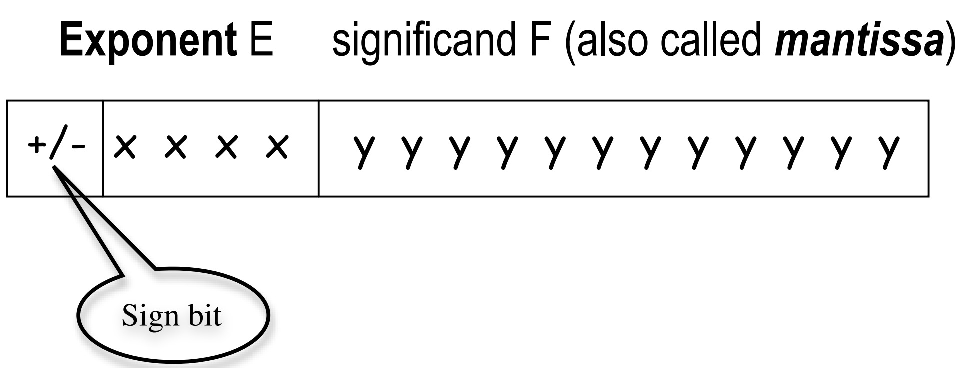
Floating point Representation of Numbers

FP is useful for representing a number in a wide range: **very small** to **very large**. It is widely used in the scientific world. Consider, the following FP representation of a number



In **decimal** it means (+/-) **1**. yyyyyyyyyyyy $\times 10^{xxxx}$

In **binary**, it means (+/-) **1**. yyyyyyyyyyyy $\times 2^{xxxx}$

(The 1 is implied)

IEEE 754 single-precision (32 bits)

s	xxxxxxxx	yyyyyyyyyyyyyyyyyyyyyyy
1	8	23 bits

Single precision

Largest = 1. 1 1 1 … x 2^{+127} ≈ 2 x 10^{+38}

Smallest = 1.000 … x 2^{-128} ≈ 1 x 10^{-38}

These can be positive and negative, depending on s.

(But there are exceptions too)

IEEE 754 double precision (64 bits)

S	exponent	significand
1	11 bits	52 bits

Largest = 1. 1 1 1… x 2^{+1023}

Smallest = 1.000… X 2^{-1024}

Overflow and **underflow** in FP

An **overflow** occurs when the number if too large to fit in the frame. An **underflow** occurs when the number is too small to fit in the given frame.

How do we represent zero?

IEEE standards committee solved this by making **zero** a special case: if every bit is zero (the sign bit being irrelevant), then the number is considered zero.

Then how do we represent 1.0?

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It should have been 1.0×2^0 (same as 0)! The way out of this is that the interpretation of the exponent bits is not straightforward. The exponent of a single-precision float is **"shift-127" encoded** (**biased representation**), meaning that the actual exponent is (xxxxxxx minus 127). So thankfully, we can get an exponent of zero by storing 127.

Exponent = 11111111 (i.e. 255) means 255-127 = 128

Exponent = 01111111 (i.e. 127) means 127-127 = 0

Exponent = 00000001 (i.e. 1) means 1-127 = -126

More on Biased Representation

The consequence of shift-127

Exponent = 00000000 (reserved for 0) can no more be used to represent the smallest number. We forego something at the lower end of the spectrum of representable exponents, (which could be 2^{-127}). That said, it seems wise, to give up the smallest exponent instead of giving up the ability to represent 1 or zero!

More special cases

Zero is not the only "special case" float. There are also representations for positive and negative infinity, and for a not-a-number (NaN) value, for results that do not make sense (for example, non-real numbers, or the result of an operation like infinity times zero). How do these work? A number is infinite if every bit of the exponent is 1 (yes, we lose another one), and is NaN if every bit of the exponent is 1 plus any mantissa bits are 1. The sign bit still distinguishes +/-inf and +/-NaN. Here are a few sample floating point representations:

Exponent	**Mantissa**	**Object**
0	0	Zero
0	Nonzero	Denormalized number*
1-254	Anything	+/- FP number
255	0	+ / - infinity
255	Nonzero	NaN like 0/0 or 0x inf

* Any non-zero number that is smaller than the smallest normal number is a denormalized number. The production of a denormal is sometimes called gradual underflow because it allows a calculation to lose precision slowly when the result is small.

Floating point operations in MIPS

32 separate single precision FP registers in MIPS

f0, f1, f2, … f31,

Can also be used as 16 double precision registers

f0, f2, f4, f30 (f0 means f0,f1 f2 means f2,f3)

These reside in a **coprocessor** C1 in the same package

Operations supported

add.**s**	\$f2, \$f4, \$f6	# f2 = f4 + f6 (single precision)
add.**d**	\$f2, \$f4, \$f6	# f2 = f4 + f6 (double precision)

(Also subtract, multiply, divide format are similar)

lwc1	\$f1, 100(\$s2)	# f1 = M [s2 + 100] (32-bit load)
mtc1	\$t0, \$f0	# f0 = t0 (move to coprocessor 1)
mfc1	\$t1, \$f1	# t1 = f1 (move from coprocessor 1)

Sample program

Evaluation of a Polynomial a.x^2 + b.x + c

Pseudo-instruction

```
        # $f0 --- x
        # $f2 --- sum of terms
        . . . . .

        # Evaluate the quadratic
        l.s         $f2,a           # sum = a
        mul.s       $f2,$f2,$f0     # sum = ax

        l.s         $f4,b           # get b
        add.s       $f2,$f2,$f4     # sum = ax + b
        mul.s       $f2,$f2,$f0     # sum = (ax+b)x = ax^2 + bx

        l.s         $f4,c           # get c
        add.s       $f2,$f2,$f4     # sum = ax^2 + bx + c
        . . . . . .

        .data
a:      .float  1.0
b:      .float  1.0
c:      .float  1.0
```

Floating Point Addition

Example using decimal

$A = 9.999 \times 10^{1}$, $B = 1.610 \times 10^{-1}$, $A+B =?$

Step 1. Align the smaller exponent with the larger one.

$B = 0.0161 \times 10^{1} = 0.016 \times 10^{1}$ (round off)

Step 2. Add significands

$9.999 + 0.016 = 10.015$, so $A+B = 10.015 \times 10^{1}$

Step 3. Normalize

$A+B = 1.0015 \times 10^{2}$

Step 4. Round off

$A+B = 1.002 \times 10^{2}$

Now, try to add 0.5 and –0.4375 in binary.

Floating Point Multiplication

Example using decimal

$A = 1.110 \times 10^{10}$, $B = 9.200 \times 10^{-5}$ $A \times B = ?$

Step 1. Exponent of $A \times B = 10 + (-5) = 5$

Step 2. Multiply significands

$1.110 \times 9.200 = 10.212000$

Step 3. Normalize the product

$10.212 \times 10^5 = 1.0212 \times 10^6$

Step 4. Round off

$A \times B = 1.021 \times 10^6$

Step 5. Decide the sign of $A \times B$ $(+ \times + = +)$

So, $A \times B = + 1.021 \times 10^6$