Class example from 11/5/2004

Show ⊢ {A>B ∧ B>0} A:= A+B; B:= A-B {A>B ∧ B>0}

Step 0: formulate Q so that

⊢ {A>B ∧ B>0}
    A:= A+B;
      {Q}
    B:= A-B
  {A>B ∧ B>0}

Based on intuitive understanding of the code, we take Q = A>2B ∧ B>0.

Step 1: show

⊢ {A>B ∧ B>0}
    A:= A+B;
      {Q}

Q[A ← A+B] = A+B>2B ∧ B>0 ≡ A>B ∧ B>0 so by the axiom of assignment, Step 1 is established.

Step 2: show

    ⊢ {B>0 ∧ A>B}
        B:= A-B
      {A>B ∧ B>0}

{A>B ∧ B>0}[B ← A-B] = A>A-B ∧ A-B>0 ≡ B>0 ∧ A>B so by the axiom of assignment, Step 2 is established.

Step 3: Q ⇒ B>0 ∧ A>B so by Step 2 and strengthening the pre-condition

    ⊢ {Q}
      B:= A-B
    {A>B ∧ B>0}

Step 4: By steps 1 and 3 and the inference rule for sequential execution,
⊢ {A>B ∧ B>0} A:= A+B; B:= A-B {A>B ∧ B>0} is established.