Home work 8 sample solution

22C:034 Spring 2004

Q1:
(a) $\neg R$
(b) $R \wedge H$
(c) $R \vee H$
(d) $R \Rightarrow H$
(e) $H \Rightarrow R$

Q2:
(c) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$
It's tautology. The truth table is in the following:

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R)$	$P \Rightarrow R$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

$((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$
T
T
T
T
T
T
T
T

(e) $(Q \wedge (P \Rightarrow Q)) \Rightarrow P$
It's contingent. The truth table is in the following:

P	Q	$P \Rightarrow Q$	$Q \wedge (P \Rightarrow Q)$	$(Q \wedge (P \Rightarrow Q)) \Rightarrow P$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

(f) $\neg(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$

It's a contradiction. The truth table is in the following:

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$\neg(P \vee (Q \wedge R))$	$P \vee Q$
T	T	T	T	T	F	T
T	T	F	F	T	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	F	T
F	T	F	F	F	T	T
F	F	T	F	F	T	F
F	F	F	F	F	T	F

$P \vee R$	$(P \vee Q) \wedge (P \vee R)$	$\neg(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$
T	T	F
T	T	F
T	T	F
T	T	F
T	T	F
F	F	F
T	F	F
F	F	F

Q3:
The full conjunctive normal form for f is:
$(P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R)$

Q4:
Proof:
1. $P \Rightarrow Q$ $premise$
2. $Q \Rightarrow R$ $premise$
3. $R \Rightarrow P$ $premise$
4. $Q \Rightarrow P$ $2, 3, HS$
5. $P \Leftrightarrow Q$ $1, 4, Equivalenceintroduction$

Q5:
proof:
1.$P \wedge Q$ $premise$
2.Q $1, Lawofsimplification$
3.$P \vee Q$ $2, variantofLawofaddition$