# Home work 4 sample solution

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Q1:

a) $P^{\sim}$ , child
$P^{\sim} \circ P^{\sim}$ , grandchild
$P \circ P$ , grandparent

b) nephew or niece

c) $(P^{\sim})^{+} \circ P^{+}$

Q2:

$f_1$ = (a,0),(b,0),(c,0)
$f_2$ = (a,0),(b,0),(c,1)
$f_3$ = (a,0),(b,0),(c,2)
$f_4$ = (a,0),(b,1),(c,0)
$f_5$ = (a,0),(b,1),(c,1)
$f_6$ = (a,0),(b,1),(c,2) one-to-one onto
$f_7$ = (a,0),(b,2),(c,0)
$f_8$ = (a,0),(b,2),(c,1) one-to-one onto
$f_9$ = (a,0),(b,2),(c,2)
$f_{10}$ = (a,1),(b,0),(c,0)
$f_{11}$ = (a,1),(b,0),(c,1)
$f_{12}$ = (a,1),(b,0),(c,2) one-to-one onto
$f_{13}$ = (a,1),(b,1),(c,0)
$f_{14}$ = (a,1),(b,1),(c,1)
$f_{15}$ = (a,1),(b,1),(c,2)
$f_{16}$ = (a,1),(b,2),(c,0) one-to-one onto
$f_{17}$ = (a,1),(b,2),(c,1)
$f_{18}$ = (a,1),(b,2),(c,2)
$f_{19}$ = (a,2),(b,0),(c,0)
$f_{20}$ = (a,2),(b,0),(c,1) one-to-one onto
$f_{21}$ = (a,2),(b,0),(c,2)
$f_{22}$ = (a,2),(b,1),(c,0) one-to-one onto
$f_{23}$ = (a,2),(b,1),(c,1)
$f_{24}$ = (a,2),(b,1),(c,2)
$f_{25}$ = (a,2),(b,2),(c,0)
$f_{26}$ = (a,2),(b,2),(c,1)
$f_{27}$ = (a,2),(b,2),(c,2)

Q3:

(a) partial function, into
(b) total function, onto, bijective
(c) no function, (a,b) and (a,a)

(d) total function, into, not bijective
(e) partial function, into

Q4:
Hint: Every real number $r$ in interval $[0,1]$ can be expressed as :

$$r = a_0 \cdot 2^{-1} + a_1 \cdot 2^{-2} + \cdots + a_{n-1} \cdot 2^{-n} + \cdots, a_i \in \{0,1\} \tag{1}$$

We can construct a function $f$ from $[0,1]$ into $P(N)$ .

$$f(r) = \{n | a_n = 1, where\ \ a_n\ \ is\ \ the\ \ a_n\ \ in\ (1), n \in N\} \tag{2}$$

Prove that $f$ is bijective.

Q5:

$$H_n = 1 + n + H_{n-1} \tag{3}$$

We'll use the idea of difference operator $\Delta$ to solve the equation. According to (3) , one has:

$$\Delta H(n) = H_{n+1} - H_n = n + 2 \tag{4}$$

We know $H(n)$ must be a quadratic. Let $H(n) = an^2 + bn + c$ . Since

$$\Delta(an^2 + bn + c) = 2an + a + b \tag{5}$$

One has:

$$\begin{cases} \text{2a=1} \\ \text{a+b=2} \end{cases}$$

So one has $a = \frac{1}{2}, b = \frac{3}{2}$ .
When $n = 0$, one has $a \cdot 0^2 + b \cdot 0 + c = H_0$ . So $c = H_0$. Consequently, one has $H_n = \frac{1}{2}n^2 + \frac{3}{2}n + H_0$ .