22c:181 Spring 2006
Homework #6 Solution

1a. To show that $\triangle$ is an equivalence relation, we must show that it is i) reflexive, ii) symmetric, and iii) transitive.

i) To show reflexivity, it must be shown that m$\triangle$m holds. Since the set of digits in m is obviously the same as the set of digits in itself, $\triangle$ is reflexive.

ii) To show symmetry, it must be shown that if m$\triangle$n holds, n$\triangle$m must also hold. If m$\triangle$n holds, then m and n have the same set of digits appearing in them, therefore, n$\triangle$m must hold.

iii) To show transitivity, it must be shown that if m$\triangle$n holds, and n$\triangle$o holds, m$\triangle$o must hold. Since m$\triangle$n, the sets of digits appearing in m and n are the same. Since n$\triangle$o, the sets of digits appearing in n and o are the same. Therefore, the sets of digits appearing in m and o are the same, and m$\triangle$o holds.

b. The equivalence classes of Nat under $\triangle$ are the members of the power set of the digits 0-9. Since $\triangle$ compares the sets of digits appearing in members of Nat, equivalence classes will be

$\triangle$[0] = {0}
$\triangle$[1] = {1, 11, 111, ...}
...
$\triangle$[0123456789] = {1023456789, 1023456798, ...}

c. $\triangle$ is not a congruence for addition. For example, m = 15, n = 51, j = 10, k = 100. m$\triangle$n and j$\triangle$k, but m+j = 25 and n+k = 151. Clearly, 25$\triangle$151 does not hold, so m+j$\triangle$n+k does not hold, so $\triangle$ is not a congruence for addition.

See website for Miranda solutions to 2 and 3.