Problem 1:
(a) $\{\varepsilon, bb, bbbb\}$
(b) $\{\text{zero or even number of 'b's}\}$
(c) $\{\text{odd number of 'b's, or aba followed by zero or even number of 'b's}\}$
(d) $S^*$
(e) $S^*$

Problem 2:
(a) $\{a\}^*$
(b) $S$
(c) $R$
(d) $\{a\}^*$

Problem 3:
(a) $(aaa)^*|(aaaaa)^*$
(b) $(aaa*b^*)|(bbb*a^*)$

Let $L_{11} = \{\text{Strings that begin with 'aa' and have no 'a' following a 'b'}\}$
$L_{12} = \{\text{Strings that begin with 'bb' and have no 'b' following an 'a'}\}$
$L_1 = \{\text{Strings that either (i) begin with 'aa' and have no 'a' following a 'b', or (ii) begin with 'bb' and have no 'b' following an 'a'}\}$
$L_{21} = L((aaa*b^*)|(bbb*a^*))$, which denotes the set of strings described by the regular expression
$L_{22} = L(bbb*a^*)$
$L_2 = L((aaa*b^*)|(bbb*a^*))$

Then $L_1 = L_{11} \cup L_{12}$, $L_2 = L_{21} \cup L_{22}$

We'll show that $L_{11} = L_{21}$.

**Proof:** If $s \in L_{11}$, then $s$ begins with 'aa' and have no 'a' following a 'b'.
Then $s = aa \cdot t$, where $t$ is a string that has no 'a' following a 'b'. So $t$ can be described by $a^*b^*$. So $s$ can be described by $aaa*b^*$, then $s \in L_{21}$, then $L_{11} \subseteq L_{21}$.

If $s \in L_{21}$, then obviously $s$ begins with 'aa' and have no 'a' following a 'b'.
Then $s \in L_{11}$, $L_{21} \subseteq L_{11}$.

Therefore, we have $L_{11} = L_{21}$. Similarly we can show that $L_{12} = L_{22}$, then we have $L_1 = L_2$.

(c) $(aa|a)(aaa)^*$
Problem 4:
(a) Yes
\[ X \Rightarrow a^3 X a \]
\[ \Rightarrow a^5 X a^3 \]
\[ \Rightarrow a^7 X a^5 \]
\[ \Rightarrow a^7 b a^5 \]

(b) No. Because the total number of 'a's in the string in \( L(X) \) should be multiple of 4.

(c) Yes
\[ X \Rightarrow a^3 X a \]
\[ \Rightarrow a^6 X a^2 \]
\[ \Rightarrow a^8 X a^4 \]
\[ \Rightarrow a^8 b a^4 \]

Problem 5:
(a) Yes
\[ X \Rightarrow X a X b X \]
\[ \Rightarrow a X b X \]
\[ \Rightarrow a X a X b X b X \]
\[ \Rightarrow a a X b X b X \]
\[ \Rightarrow a a b X b X \]
\[ \Rightarrow a a b X a X b X \]
\[ \Rightarrow a a b b a X b X \]
\[ \Rightarrow a a b a a X b X \]
\[ \Rightarrow a a b a b a X b X \]
\[ \Rightarrow a a a b b a b a X b X \]
\[ \Rightarrow a a a b b a b b \]

(b) No. Because number of 'a's \( \neq \) number of 'b's.

(c) No. Because for any string in \( L(X) \), at any place in the string, the number of 'a's before should \( \geq \) the number of 'b's before. (This statement can be proved by induction.) But in this string as it’s marked, \( abb \mid aab \), number of 'b's = 2, number of 'a's = 1 at the mark.

(d) Yes
\[ X \Rightarrow X a X b X \]
\[ \Rightarrow a X b X \]
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