Homework 10
sample solutions

Problem 1
(a) $x-y+1$ -- this is greater than 0 when $x \geq y$
(b) $1-x^2$ -- this is greater than 0 only when $x=0$
(c) $x^2$ -- this is greater than 0 whenever $x \neq 0$

Executable versions of solutions for problems 2 & 3 are in the class directory.

Problem 4.
Step 1: discover the loop invariant
The loop invariant in conjunction with the negation of the loop guard must imply the post-condition. So the "difference" between these conditions is a general guide for what is needed for the loop invariant. In this case the loop invariant $\equiv n>(\sqrt{}-1)^2$.

Step 2: prove invariant true at first arrival -- $\{n \geq 1\}$ $\sqrt{} := 1$ $\{\text{loop invariant}\}$
by the Axiom of assignment
$\{ n>0 \}$
$\sqrt{} := 1$
$\{ n>(\sqrt{}-1)^2 \}$

Step 3: prove this assertion is "invariant" -- still true after the execution of the loop body, given the loop guard.
3A. by the Axiom of Assignment
$\{ n>\sqrt{}^2 \}$
$\sqrt{} := \sqrt{}+1$
$\{ n>(\sqrt{}-1)^2 \}$
3B. by Strengthening the pre-condition in Step 3A, Step 3 is proven since $n>\sqrt{}^2 \land n>(\sqrt{}-1)^2 \implies n>\sqrt{}^2$

Step 4: prove the While post-condition
by the While rule on Step 3
$\{\text{loop invariant}\}$
$\{\text{loop invariant} \land n \leq \sqrt{}^2 \}$

Step 5: prove the program
by the Sequential execution rule on Steps 2 and 4, at the conclusion of the program the post-condition $\{ \text{loop invariant} \land n \leq \sqrt{}^2 \} \Rightarrow \{ n>(\sqrt{}-1)^2 \land n \leq \sqrt{}^2 \}$ is proven.